Decentralized Control of Interconnected Dynamical Systems

by

Isam Esa Abdul-Majid Al-Abdullah

A Thesis Presented to the

FACULTY OF THE COLLEGE OF GRADUATE STUDIES

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In Partial Fulfillment of the
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This thesis, written by

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بسم الله الرحمن الرحيم

IN THE NAME OF ALLAH, THE MOST
MERCIFUL AND MOST GRACIOUS

This thesis is dedicated to my dear parents, whose anxiety, patience and encouragement enabled me to complete this research

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ABSTRACT

A decentralized model-following approach to design a suboptimal controller for large interconnected dynamical systems is considered. The system is assumed to be subjected to an input disturbance. Interactions between subsystems are represented by a reduced order model. Gain parameters of the decentralized controller are independent of the initial conditions.

A modified decentralized model-following controller is also developed and compared with the previous approach through an illustrative example. The implementation of the developed controller is simpler than the first approach. The resulting controller can be easily implemented using microprocessors.

A decentralized interaction-rejection approach is also considered. A practical industrial system controlling the thickness of a slab in a hot rolling finishing mill is considered. Decentralized controllers using the previous approaches are developed to maintain variations of slab thickness as small as possible.

1. INTRODUCTION

1.1 OVERVIEW

Centralized control of large interconnected systems is not appealing nowadays due to expensive cost of communication between the distributed communication networks and extensive computational requirements. In many applications, it may be difficult to implement the resulting controller.

Other approaches may be employed to overcome these problems, but not all of them are practical. Among these approaches are aggregation, multilevel control and decentralized control.

Using aggregation, the designer attempts to find a reduced order model to represent the original system. The controller is then designed based on this low order model, not the original system [1,2,3].

Design and analysis are carried out on the aggregated model with less computational expense but the validity of the results depends on the accuracy of the approximation. The validity of this approximation in turn depends on the system and its inputs as well as the aggregation technique used. Also the aggregated model cannot correctly predict the response to an initial condition disturbance heavily concentrated in a mode that is omitted. Thus the expected initial condition disturbance must be taken in consideration in the selection of an aggregated model.

If the aggregated model is used to determine a simplified feedback structure, the situation becomes more complex. Modes that are neglected in the aggregated model may destabilize the system [4].

Since the design of a good centralized controller for a large scale interconnected system is a difficult task, therefore it would be easy to consider a set of subsystems of low order by decomposing the large scale system to a number of small order subsystems. This is usually based on geographical, physical or mathematical basis.

After the decomposition of the large interconnected system, the design of the controller for each subsystem will be easier, but this is insufficient to achieve the global control of the whole system. That is due to the interactions between the subsystems. Therefore there is a need to consider interactions between subsystems. It can be achieved using a coordinator at a higher level which will handle the communication between the controllers of the subsystems to achieve the global optimal control. This approach is referred as multilevel control [4-12].

In the implementation of this control, the communication links between the controllers and the coordinator are the major cost. Also the system may lose its stability due to unexpected cut of the communication links between the subsystems and the coordinator [4,5].

Some of the approaches in the multilevel control depend on the initial values of the states of the system and any change in the initial values needs another design of the controller [5].

In case of weak coupling between the subsystems, the interaction between the subsystems can be ignored. A decentralized controller can be easily developed and implemented using microprocessors.

While, in case of strong coupling between the subsystems, the effect of the interaction with decentralized control can be taken into account by using certain techniques such as model following and disturbance rejection which will be discussed in this thesis afterwards in details.

In the model following technique, the interactions are generated in the controller of each subsystem using a reduced order model for the interactions.

In the disturbance rejection technique, the interactions are assumed to be a disturbance, and the performance index is modified to include a control component which will compensate the disturbance [8,9].

1.2 OBJECTIVES

The objectives of this thesis can be outlined as follows:

1. The model following approach for decentralized control of linear interconnected dynamical system proposed in [6,7] is considered.

   The technique will be extended for linear systems subjected to a known input disturbance.

2. A modified interaction model following decentralized control will be developed for linear systems subjected to a known input disturbance.

   A model following decentralized controller is also developed for linear systems subjected to unknown constant input disturbance.

3. A practical industrial system controlling the thickness of a slab in a hot rolling finishing mill will be considered. Decentralized controllers will be developed to maintain the variations in the slab thickness as small as possible.

1.3 THESIS OUTLINE

This thesis consists of six chapters. Chapter 1 serves as a general introduction to the control of large scale systems. It also includes the objectives and the outline of the thesis.

The literature review of the subject is included in Chapter 2.

In Chapter 3, the model following approach of decentralized control for linear interconnected dynamical system is presented.

In Chapter 4 the modified model following interaction prediction decentralized controller is developed and compared with the previous approaches through illustrative example. The disturbance rejection approach proposed in [8,9] is also investigated.

In Chapter 5 both approaches presented in Chapters 3 and 4 are applied to a practical example of a hot rolling finishing steel mill. The results of the simulation are compared with the optimal controller.

A general conclusion of the thesis work and future suggestions for further research are included in Chapter 6.

2. LITERATURE REVIEW

2.1 OVERVIEW

Physical dynamical systems are in general, described by non-linear differential equations. It is often possible to linearize the non-linear equation around the operating point, to develop a linear model. Therefore, many systems could be described by the following state space model:

$$\dot{x}(t) = A\,x(t) + B\,u(t) + E\,d(t)$$
$$x(t_o) = x_o \tag{2.1}$$

where x(t) is the state vector of the system and u(t) is the control input vector and d(t) is a known disturbance. The control input u(t) may be determined to minimize the performance index:

$$J = \frac{1}{2}\; x_f^T\, H\, x_f \;+\; \frac{1}{2}\int_o^{\infty} x^T Q\, x\, dt + \frac{1}{2}\int_o^{\infty} u^T R\, u\, dt \tag{2.2}$$

where R is positive definite weighting matrix, Q and H are semipositive weighting matrices. Without loss of generality, Q, R, and H are considered as symmetric matrices.

The optimal feedback control law takes the form:

$$u(t) = K(t)\,x(t) + v(t) \tag{2.3}$$

where

$$K(t) = -R^{-1} B^T P(t)$$

$$v(t) = -R^{-1} B^T S(t)$$

P(t) is the solution of the Ricatti equation

$$\dot{P}(t) = -P(t) A - A^T P(t) + P(t) B R^{-1} B^T P(t) - Q$$

$$P(t_f) = H \tag{2.4}$$

and s(t) is the solution of

$$\dot{s}(t) = -[A - B R^{-1} B^T P(t)]^T s(t) + P(t) E d(t)$$

$$s(t_f) = H \tag{2.5}$$

For large scale systems, i.e. n is large, the solution of (2.4) necessitates the solution of n(n+1)/2 differential equations. This requires large computer storage and computational time.

In this case, the design of controller can be performed either by developing an aggregated model of low dimension [2], or by decomposing the original system into a number of subsystems and developing a multilevel control scheme [10] as in Fig. 2.1.

Although the computation of the optimal control for large scale interconnected dynamical systems can be performed with a multilevel structure [10], the implementation of the optimal controllers raises certain difficulties.

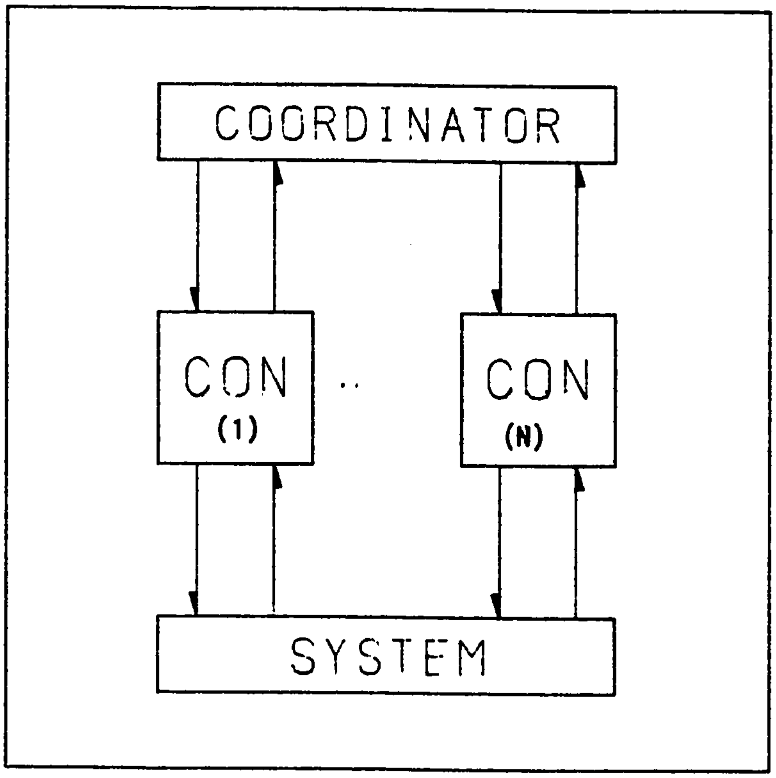

Figure 2.1. Multilevel control structure.

In particular, for systems where the subsystems are geographically distributed, the feedback of state information between the subsystems controllers can prove to be the major component of the cost in implementation phase. Examples of large scale systems include:

(a) power systems,

(b) urban traffic systems,

(c) digital communications systems,

(d) flexible manufacturing systems,

(e) ecological systems,

(f) economical systems,

(g) steel industrial systems.

Robust decentralized controllers [8], where the transfer of states information from other subsystems is not needed, seem to be very appealing to control engineers, Fig. 2.2.

The linear interconnected dynamical system is represented by a set of subsystems in the following form:

$$\dot{x}_i(t) = A_i x_i + B_i u_i + C_i z_i + E_i d_i$$
$$x_i(t_o) = x_{i\,o} \qquad i = 1, 2, \dots, N \qquad (2.6)$$

where z_i represents the interconnection variable

$$z_i = \sum_{j \neq i}^{N} L_{ij} x_j$$

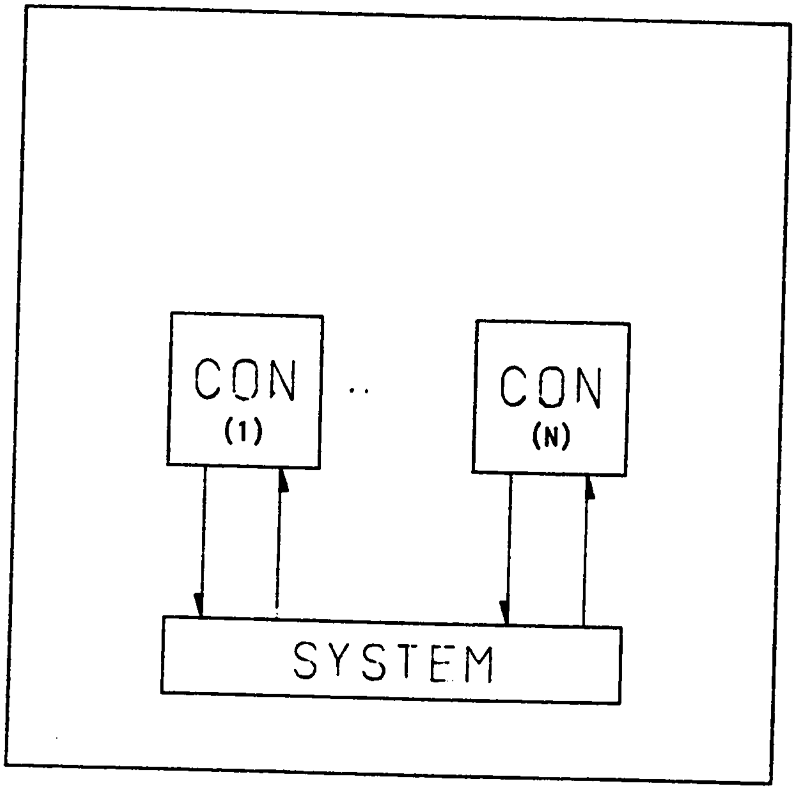

Figure 2.2. Decentralized control structure.

where $d_i(t)$ is a known disturbance input. The control input $u_i(t)$ may be determined to minimize the performance index:

$$J_i = \frac{1}{2} [\int_0^{\infty} (x_i^T \; Q \, x_i + u_i^T R \, u_i) \, dt] \qquad (2.7)$$

where $$J = \sum_{i=1}^{N} J_i$$

R_i is positive definite weighting matrix, Q_i is semipositive weighting matrices. Without loss of generality, H is assumed zero in (2.2).

Robust decentralized controllers for large scale systems may be needed due to:

(1) either the lack of centralized information,

(2) or the lack of centralized computing capability.

In case of a weakly coupled system, the interaction between the subsystems can be neglected and the resulting decentralized control proved to be effective [4]. In case of when the effect of interaction will be taken into account then certain techniques must be used such as model following technique.

2.2 MULTILEVEL CONTROL

The basic idea of the multilevel control scheme is to deal with several small subsystems instead of large integrated system.

This involves two phases, the first phase is to decompose the large integrated system to a set of small subsystem at the lower level while in the second phase coordinator is required to coordinate between subsystems at higher levels.

In the lower level, the controller manipulates a set of variables which is called local variables. In the higher levels, the control manipulates another set of variables which is called coordinating variables. The task of the higher levels is to manipulate the coordinating variables in such a way so that the independent lower level subsystems are forced to reach an overall system optimal states and control [10,11].

2.2.1 The Model Coordination Approach

It converts the control to two levels and fixes the interconnection variables at some value in first level, then tries to optimize with respect to the interconnection variables in the second level.

The structure of this multilevel scheme is depicted in Fig. 2.3 [11], where x_i is state variables vector and z_i is interaction variables vector.

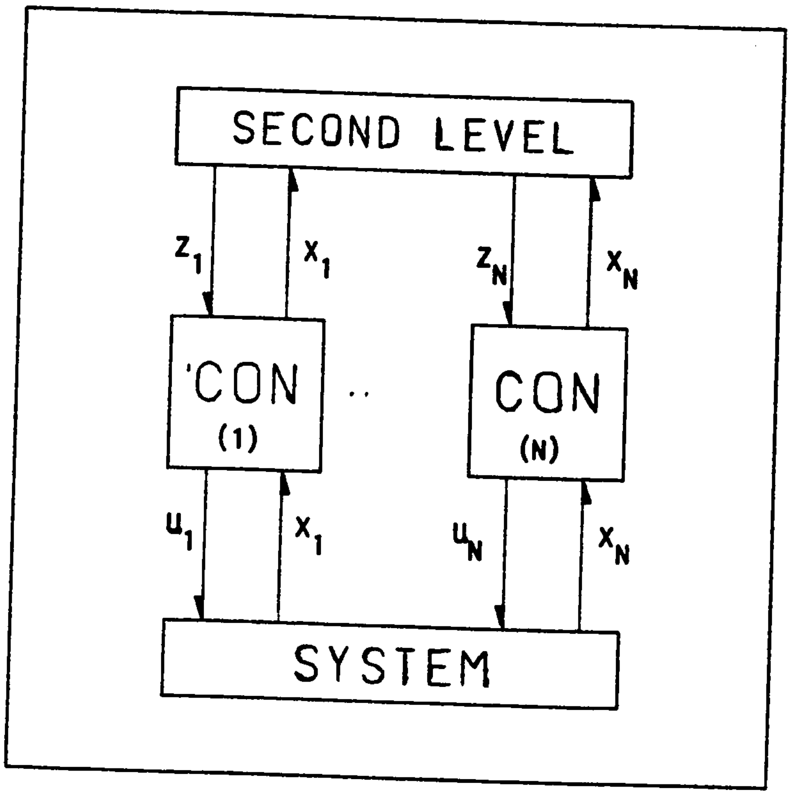

Figure 2.3. Multilevel structure of model coordination.

2.2.2 The Goal Coordination Approach

Using this approach, it is necessary to remove the interconnection among subsystems. Considering the addition of a penalty term which penalizes the performance of the system if interconnection variables do not balance using a vector of weighting parameters which are called Lagrange multipliers (λ_i).

The structure of this multilevel scheme is depicted in Fig. 2.4 [11].

2.2.3 Other Approaches

Tammura [11] made an interesting modification to the Goal Coordination approach which made it more attractive. The modification is to add an additional level to the lower stage, in which the optimization is done at a certain instant of time $k = k_i$ in each subproblem, where the system states are treated as parameters.

The interactions (z_i) between the subsystems are calculated at the higher level using interaction prediction method in addition to the costate vector (i.e. the Lagrange multipliers λ_i's are calculated at the coordination level. It is obvious that there are more calculations in the coordination level than the Goal Coordination Approach. This reduces the complexity and the calculation in lower level.

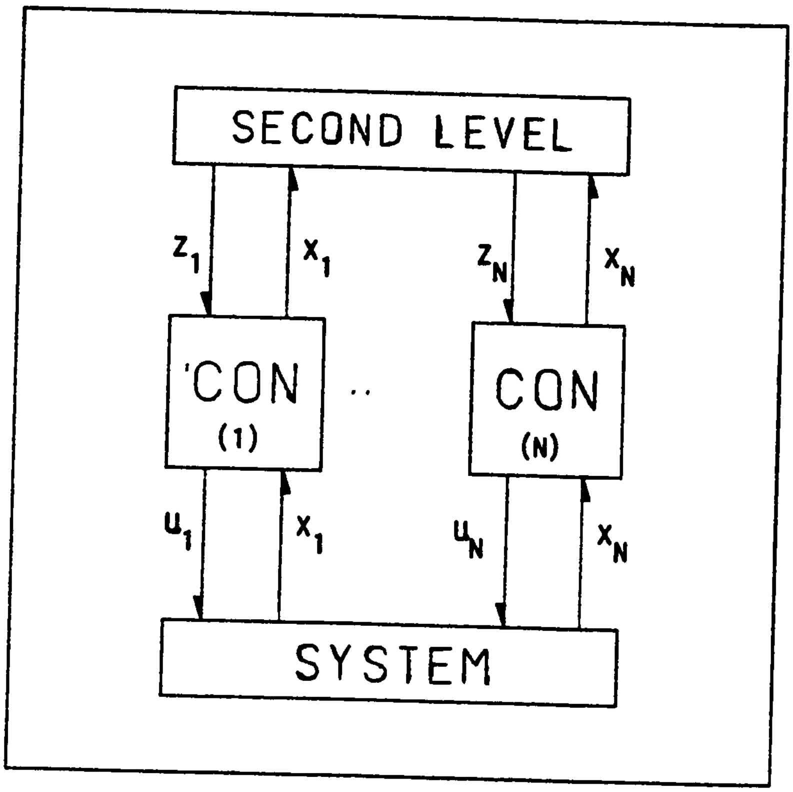

Figure 2.3. Multilevel structure of model coordination.

A multilevel control approach is developed by Siljak and Sundareshan in [18] where for local controllers large-scale linear continuous dynamic systems are designed to optimize the performance measure of each subsystem, ignoring the interconnections. Then the coordinator is developed to minimize the effect of interconnections and improve the performance of the overall system. Aly et al [19,20] have used this approach for load frequency control of linear interconnected power system. The resulting controller showed to be very effective in this application.

Fawazy, Hinton, and Hana [12] introduced a three level approach where the third level updates a weighting factor in the performance measure for non-linear system.

2.3 DECENTRALIZED CONTROL

During recent years, there has been increasing interest in the study of decentralized control of large-scale multivariable system using local controllers. Each controller observes only the local systems states outputs to produce local system control inputs. This is done due to the expensive cost of the coordination and the communications links.

The advantages of the decentralized control can be outlined as follow:

1. The finite time of computation.
2. The decreasing complexity of the control system.

3. Ease of design because of the decomposition of the large scale system to a set of small subsystems.
4. Parallel operation and computation in the local controllers.
5. System reliability because the failure of local controller effects the subsystem only.
6. Using microcomputer and microprocessor to implement the decentralized controllers.
7. Reducing the cost of the control system.

2.3.1 Weak Coupled System

The decentralized approach in case of weak coupled systems is to neglect the interaction between the subsystems so that the A and B matrices will be block diagonal, i.e.

Assume we have the state equation of the system in this form

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} \qquad (2.8)$$

where A_{12}, A_{22} are very small with respect to A_{11}, A_{22}.

Then the state equations can be rewritten in this form:

$$\begin{bmatrix} \dot{x}_1(t) \\ \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} A_{11} & 0 \\ \\ 0 & A_{22} \end{bmatrix} \begin{bmatrix} x_1(t) \\ \\ x_2(t) \end{bmatrix} + \begin{bmatrix} B_1 & 0 \\ \\ 0 & B_2 \end{bmatrix} \begin{bmatrix} u_1(t) \\ \\ u_2(t) \end{bmatrix} \qquad (2.9)$$

By this way, computation in the simulation, control system design and implementation of the control is reduced since the calculation is done on two independent lower dimension subsystems. Moreover, control systems designed for the decoupled subsystems provide a completely decentralized control structure for the original system, as illustrated in Fig. 2.2 [4]. In [13], an explicit weak coupling condition is obtained which insures that the approximate solution stabilizes the actual system. This condition is always satisfied for small interaction.

2.3.2 Strong Coupled Systems

In this type of systems, the interaction can not be ignored.

Hassan and Singh [8] developed an entirely different approach for the design of robust decentralized controller for linear interconnected dynamical systems. A simple reduced order model for interactions is introduced to compute controller parameters. Unknown constant external disturbances can be also accommodated.

In [9], the computation of the control is done subsystem by subsystem so that it is not necessary to solve the global problem off-line within an hierarchical structure as in [8]. This makes the design of the local controllers so simple.

The output of the interaction model is improved online using a model-following technique in [7], and the controller gains are calculated only once, and they are easy to compute since all operations are done at subsystem level. The algorithm is modified in [6], to tackle nonstandard problems by choosing the dominant poles in the interaction model.

Two algorithms for decentralized control for large scale linear systems have been developed by Guangquan and Lee [14] where the approach is computationally attractive in that existing Ricatti matrix solution may be directly applied. These algorithms are very simple to implement due to this matter. It is observed that the decentralized controller works well even if there may be some small variance in the system parameters or some small external disturbance.

A simple method for controlling non-linear systems has also been developed in [15] for optimization of non-linear dynamical systems with a quadratic performance measure function. This method used an expansion around the operating point of the system to fix the second and high order terms. These terms are compensated for, iteratively, at the second level. The convergence of the method is assured and experience showed that it is quite rapid. Savings in both computation time and computer storage make the method an attractive alternative for solving non-linear problems [21].

3. DECENTRALIZED CONTROL USING INTERACTION MODELS

3.1 INTRODUCTION

In this chapter, the model following interaction prediction approach is introduced. Three techniques of model following are presented. These are the real model following, the implicit model following and the modified implicit model following. The decentralized control using model following technique which is proposed in [6,7] with a modification to accommodate a known disturbance is developed. An illustrative example is included.

3.2 MODEL FOLLOWING CONCEPTS

One of the requirements for the design of an optimal control system, is the optimum performance measure. In some cases, the performance measure is simple as in the minimum-time problem, while in many cases, it is not. In most of the cases, it is a regulator type which consists of an integral of quadratic weighted state variables error and control inputs, to be minimized to produce a controller that maintains the states at their normal values and minimum control effort.

Stating the model in linear differential equations subject to a quadratic performance measure has been widely used. Its popularity is due to its mathematical properties rather than to its accuracy in reflecting engineering requirements. A performance measure which is sometimes more realistic is one in which control is used to make the actual system behaves like a specified model. Such approach is called model following. Three techniques on the model following will be discussed [16].

3.2.1 Real Model Following [16]

In this technique, the original system equations and the dynamic equations defining the model are combined into a single system. The performance measure includes the difference between the state of the original system and that of the model. The original system equations are:

$$\dot{x} = A\,x + B\,u \tag{3.1}$$

and the model is described by:

$$\dot{x}_m = A_m\, x_m \tag{3.2}$$

Combining these equations into a single system, we get:

$$\dot{y} = \begin{bmatrix} A & 0 \\ 0 & A_m \end{bmatrix} y + \begin{bmatrix} B \\ 0 \end{bmatrix} u \tag{3.3}$$

where $y^T = [x \quad x_m]$

The performance measure for achieving model following is

$$\min_{u} J = \int_0^T \{(x-x_m)^T Q(x-x_m) + u^T R\, u\}\, dt \tag{3.4}$$

which can be written in the form

$$\min_{u} J = \int_0^T \{y^T Q' y + u\, R\, u\}\, dt \tag{3.5}$$

where $Q' = \begin{bmatrix} Q & -Q \\ -Q & Q \end{bmatrix}$

In this way, we have the model following problem in the conventional optimal-regulator format. Solving the steady-state Riccati equation, the optimal control law that results, will be in the following form:

$$u = K\, y \tag{3.6}$$

This can be divided into two parts as follows:

$$u = K_b\, x + K_f\, x_m \tag{3.7}$$

K_b represents the feedback gain.

K_f represents the feedforward gain.

The real model following is shown in Fig. 3.1.

3.2.2 Implicit Model-Following [16]

In this technique, the performance measure is selected to minimize the dynamical differences between the model and the original system as stated below:

$$\min J = \int_o^T (A\,x + Bu - A_m x)^T Q(A\,x - A_m x) + u^T R\,u\}\,dt \qquad (3.8)$$

subject to $\dot{x} = A\,x + B\,u$.

In this case, the optimal control law will be:

$$u = K\,x \qquad (3.9)$$

where $K = -R'^{-1}\{B^T Q(A - A_m) + B^T P\}$,

P is the solution of the steady state Riccati equation as given in equation (2.4), where,

$$R' = R + B^T Q\,B$$

3.2.3 Modified Implicit Model-Following [16]

This technique is the same as the previous one in the principle. The least squares method is used to calculate the optimal control which takes the form;

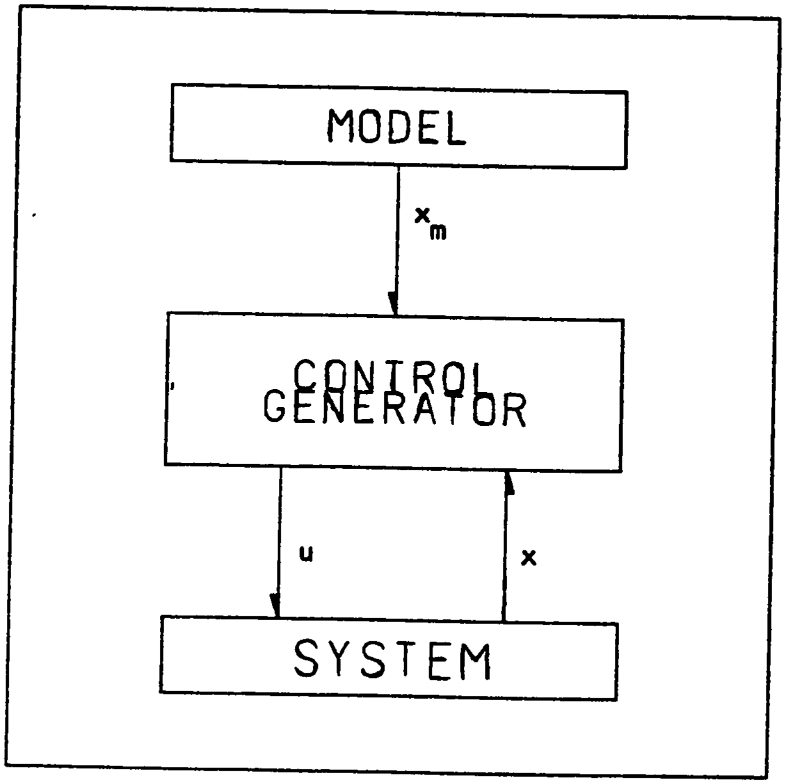

Figure 3.1. Real model following.

$$u = -(B^T B)^{-1} B^T (A - A_m) x \tag{3.10}$$

where Q and R matrices are assumed unity matrices.

An extended result can be obtained for a weighted least squares method so that the optimal control law will be

$$u = -(B^T Q B)^{-1} B^T Q (A - A_m) x \tag{3.11}$$

assuming R is a unity matrix.

3.3 DECENTRALIZED CONTROL USING MODEL FOLLOWING TECHNIQUE

Hassan and Singh [6,7] have developed an approach based on model-following, in which only subsystems calculations are needed. This is done by choosing a crude model of reduced order for the interactions between the subsystems. As a result, a complete decentralized controller is achieved.

Although they claimed that their approach stands for an input disturbance, the developed controller does not include a compensating component for the disturbance. In what follows, we add a disturbance term in the subsystem model includes a disturbance term and decentralized controller is developed that will stand for a known disturbance.

3.3.1 Problem Formulation

We shall consider a dynamic system described by the Eqn. (2.1). The control input u(t) may be determined to minimize the performance measure given in Eqn. (2.2) where $H = 0$. Assuming that Q and R are diagonal matrices, the problem can be rewritten in the following decomposed form:

$$\min J = \sum_{i=1}^{N} \frac{1}{2} \int_{0}^{\infty} \{ x_i^T Q_i x_i + u_i^T R_i u_i \} \, dt \qquad (3.12)$$

Subject to:

$$\dot{x}_i = A_i x_i B_i u_i + C_i z_i + E_i d_i \qquad (3.13)$$

$$z_i = \sum_{j=1}^{N} L_{ij} \, x_j \qquad \ldots.. \, i = 1, N$$

where x_i is the n_i-dimensional state vector of the i^{th} subsystem, u_i is the m_i-dimensional control vector, z_i is the q_i-dimensional interconnection vector and d_i is r_i-dimensional known disturbance vector.

Since z_i is a function of the other subsystems state variables therefore it is not available at the subsystem level. So let us assume that z_i can be represented by the following dynamical model:

$$\dot{z}_i = A_{zi} \, z_i \qquad (3.14)$$

A_{zi} is an arbitrary matrix which describes a crude model for z_i. A_{zi} can be chosen to be the block of the global A matrix corresponding to the vector z_i.

To clarify the idea of the choice of A_{zi}, let us assume, as an example, that the global A matrix is given by:

$$\left[\begin{array}{c:cc} -3 & -1 & +1 \\ \hdashline 0 & -2 & 3 \\ 5 & 1 & -1 \end{array}\right]$$

If this system is decomposed into two subsystems along the dotted lines then:

$$\dot{x}_1 = -3\, x_1 + C_1 z_1$$

where $C_1 = [-1 \quad +1]$

$$z_1^T = [\,x_2 \quad x_3]$$

Choose $A_{z1} = \begin{bmatrix} -2 & 3 \\ 1 & -1 \end{bmatrix}$

and,

$$\dot{x}_2 = A_2\, x_2 + C_2 z_2$$

where $A_2 = \begin{bmatrix} -2 & 3 \\ 1 & -1 \end{bmatrix}$

$C_2 = [0 \quad 5]^T$

$z_2 = x_1$

Choose $A_{z2} = -3$

Another way of choosing A_{zi} may be to concentrate on the eigenvalues. However, in the multivariable case, the relationship between the eigenvalues, and the states is not so clear [6].

A simple way of specifying good response is in assignment of the poles of the model in the left halfplane. Thus, we could choose the matrix A_{zi} to be diagonal with negative elements.

Equations (3.12), (3.13) and (3.14) can be combined together to form a modified optimization problem in the following form:

$$\min J = \sum_{i=1}^{N} \frac{1}{2} \int_0^{\infty} (y_i^T Q_{ai} y_i + u_i^T R_i u_i)\, dt \tag{3.15a}$$

Subject to:

$$\dot{y}_i = A_{ai} y_i + B_{ai} u_i + D_{ai} \tag{3.15b}$$

where

$$y_i^T = [x_i \quad z_i], \qquad A_{ai} = \begin{bmatrix} A_i & C_i \\ & \\ 0 & A_{zi} \end{bmatrix}, \quad B_{ai}^T = [B_i \quad 0]$$

$$Q_{ai} = \begin{bmatrix} Q_i & 0 \\ & \\ 0 & 0 \end{bmatrix}, \quad D_{ai}^T = [B_i d_i \quad 0]$$

3.3.2 Development of Decentralized Controller

The resulting gain matrix will depend on the chosen matrix A_{zi} and on the trajectories of z_i, since the control u_i will be a function of both x_i and z_i. Now, since in most of the practical applications z_i is not available for measurement, we shall try to modify the trajectories of z_i.

Let us assume that the ith subsystem is controllable. Then, the solution of the Eqn. (3.15)

$$u_i^* = -R_i^{-1} B_{ai}^T P_i y_i + v_i \tag{3.16}$$

where P_i is the solution of steady state Riccati Eqn. (2.4)

v_i is the solution of the constant term Eqn. (2.5)

The optimal control can be written in terms of x_i and z_i as:

$$u_i^* = -G_{ix} x_i - G_{iz} z_i + v_i \tag{3.17}$$

From the above equation, we see that the gain matrix is a function of the approximate system model. Now, although we can obtain z_i from the interconnection model, this model is a crude one. It is therefore necessary to improve the model online. To do this, let us consider a subsystem model whose input z_i is provided by the crude model. To distinguish between the z_i which is the actual interaction and z_{mi} which is the interaction produced by the model, we will use different symbols i.e.,

$$\dot{x}_{mi} = A_i x_{mi} + B_i u_i + C_i z_{mi} + E_i d_i \tag{3.18}$$

where x_{mi} is the state vector of the subsystem model. Then, if we substitute for u_i in Eqns. (3.13) and (3.18), after replacing z_i by z_{mi} in Eqn. (3.17) we have

$$\dot{x}_i = A_{oi} x_i + C_i z_i - B_i G_{iz} z_{mi} + W_i \tag{3.19}$$

where $A_{oi} = A_i - B_i G_{ix}$

$W_i = E_i d_i + B_i v_i$

x_i is the suboptimal state the i<u>th</u> of subsystem resulting from using z_{mi} instead of z_i in the control, and

$$\dot{x}_{mi} = A_{oi}\, x_{mi} + C_i\, z_{mi} - B_i\, G_{iz}\, z_{mi} + W_i \tag{3.20}$$

Thus, substracting Eqn. (3.20) from (3.19) we obtain:

$$\dot{x}_{ei} = A_{ei}\, x_{ei} + C_i\, z_{ei} \tag{3.21}$$

where

$$x_{ei} = x_i - x_{mi} \tag{3.22}$$

$$z_{ei} = z_i - z_{mi} \tag{3.23}$$

and our aim to minimize the error vectors x_{ei} and z_{ei}. For this problem, we can construct another optimization problem to determine z_{ei} to achieve minimum x_{ei}, that is,

$$\min J_{ei} = \frac{1}{2} \int_0^{\infty} (x_{ei}^T\, H_i\, x_{ei} + z_{ei}^T\, S_i\, z_{ei})\, dt \tag{3.24}$$

Subject to the constraint Eqn. (3.21), where H_i and S_i are positive-semidefinite and positive-definite arbitrary matrices respectively.

The solution of this problem is

$$z_{ei} = -S_i^{-1}\, C_i^T\, K_i\, x_{ei} \tag{3.25}$$

where K_i is a solution of a steady state Riccati equation.

Using Eqns. (3.25) and (3.23), we obtain:

$$z_i^* = z_{mi} - S_i^{-1} C_i^T K_i x_{ei} \tag{3.26}$$

and we will use this to generate the control. Thus the dynamical equations for the subsystem will be:

$$\dot{x}_i = A_{oi} x_i + C_i z_i - B_i G_{iz} z_i^* + W_i \tag{3.27}$$

$$\dot{x}_{mi} = A_{oi} x_{mi} + C_i z_{mi} - B_i G_{iz} z_{mi} + W_i \tag{3.28}$$

$$\dot{z}_{mi} = A_{zi} z_{mi} \tag{3.29}$$

$$z_i^* = z_{mi} - S_i^{-1} C_i^T K_i (x_i - x_{mi}) \tag{3.30}$$

This controller is depicted in the block diagram in Fig. 3.2.

3.4 ILLUSTRATIVE EXAMPLE

Let us consider the following problem [17]. A dynamical system is described by the differential equations:

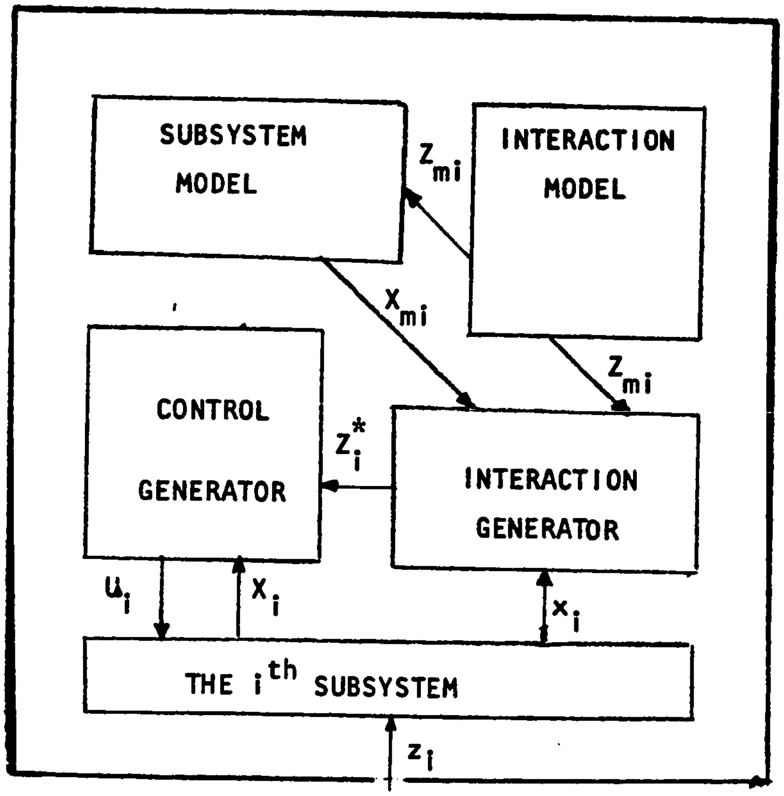

Figure 3.2. Decentralized controller using model following.

$$\dot{x}_1(t) = x_2(t) - u_1(t) + 1.0$$
$$\dot{x}_2(t) = 2x_1(t) - x_2(t) + u_2(t) + 1.0$$

It is required to determine the optimal control function u(t) for the above system while minimizing the performance measure,

$$J = \int_0^\infty (x_1^2(t) + 0.5\, x_2^2(t) + 0.5\, u_1^2(t) + 0.5\, u_2^2(t))\, dt$$

Let us rewrite these equations in the standard form:

$$\dot{x} = A\,x + B\,u + E\,d$$

where

$$A = \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \; E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \; d = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$J = \frac{1}{2} \int_0^\infty (x^T Q\, x + u^T R\, u)\, dt$$

where $Q = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}, \; R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Optimal Controller:

The optimal control takes the form

$$u = -R^{-1} B^T P x + v$$

where the Riccati matrix $P = \begin{bmatrix} 2.2042 & 0.9316 \\ 0.9316 & 0.7307 \end{bmatrix}$

Then,

$$u_1 = +2.2042 \ x_1 + 0.9316 \ x_2 + 1.9251$$
$$u_2 = -0.9316 \ x_1 - 0.7307 \ x_2 - 1.0366$$

The interaction variables z_1, z_2 represent the coupling between the two state equations. $z_1 = x_2$, and $z_2 = x_1$. Simulation results showing states trajectories, interaction variables and the control functions are given in Fig. 3.3.

Decentralized Model-Following Controller:

The system is decomposed into two subsystems. The first subsystem is represented by the state x_1:

$$\dot{x}_1 = z_1 - u_1$$
$$z_1 = x_2$$

and the second subsystem is represented by the state x_2:

$$\dot{x}_2 = -x_2 + 2\, z_2 + u_2$$

$$z_2 = x_1$$

The performance measure is also decomposed into two parts:

$$J_1 = \int_0^\infty (x_1^2 + 0.5\, u_1^2)\, dt$$

and

$$J_2 = \int_0^\infty (0.5\, x_2^2 + 0.5\, u_2^2)\, dt$$

Now, we have two subproblems. Let us choose a crude model for the interaction for each subsystem of the form:

$$\dot{z}_1 = -z_1$$

$$\dot{z}_2 = -2\, z_2$$

Putting the two subproblems in the model-following form, the first subproblem matrices are:

$$A_1 = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} -1.0 \\ 0.0 \end{bmatrix}, D_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$Q_1 = \begin{bmatrix} 2.0 & 0.0 \\ 0.0 & 0.0 \end{bmatrix}$$

$$R_1 = 1.0$$

and the second subproblem matrices are:

$$A_2 = \begin{bmatrix} -1.0 & 2.0 \\ 0.0 & -2.0 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 1.0 \\ 0.0 \end{bmatrix}$$

$$D_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$Q_2 = \begin{bmatrix} 1.0 & 0.0 \\ 0.0 & 0.0 \end{bmatrix}$$

$$R_2 = 1.0$$

Gain matrices for each subsystem are computed as:

$$G_1 = [-1.4142 \quad -0.5858]$$

$$K_1 = 0.0690$$

$$G_2 = [0.4142 \quad 0.2426]$$

$$K_2 = 0.1296$$

The equations of the first subproblem are:

$$\dot{z}_{m1} = -z_{m1}$$
$$\dot{x}_{m1} = -1.4742\, x_{m1} + 0.4142\, z_{m1}$$
$$x_{e1} = x_1 - x_{m1}$$
$$z_{e1} = -0.069\, x_{e1}$$
$$z_1 = z_{m1} + z_{e1}$$
$$u_1 = 1.4142\, x_1 + 0.5858\, z_1 + 1.0$$

where x_1 will be input to the controller from the subsystem (1) and u_1 will be output from the controller to the subsystem (1).

The equations for the second subproblem will be

$$\dot{z}_{m2} = -2\, z_{m2}$$
$$\dot{x}_{m2} = -1.4142\, x_{m2} + 1.7574\, z_{m2} + 0.7071$$
$$x_{e2} = x_2 - x_{m2}$$
$$z_{e2} = 0.1296\, x_{e2}$$
$$z_2 = z_{m2} + z_{e2}$$
$$u_2 = -0.4142\, x_2 - 0.2426\, z_2 - 0.2929$$

where x_2 will be an input to the controller from subsystem (2) and u_2 will be an output from the controller to subsystem (2).

Simulation results showing the the state trajectories, interaction variables and the control functions are given in Fig. 3.3.

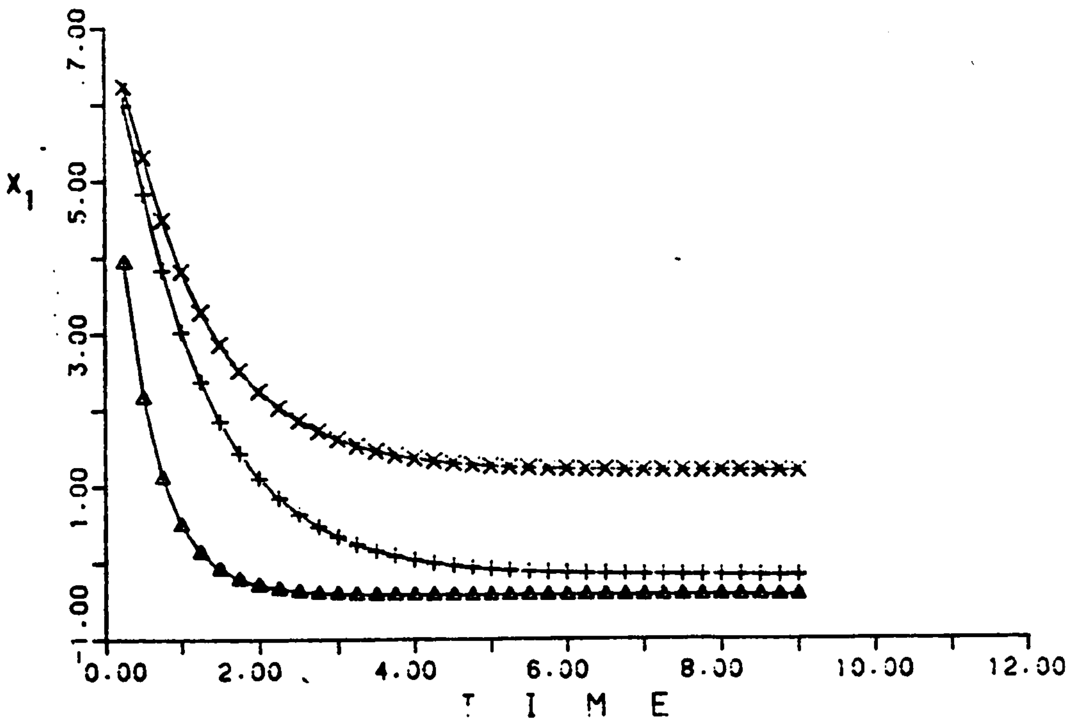

Figure 3.3(a). The state x_1 trajectory.

Δ optimal

+ developed model following

x model following as in [6,7]

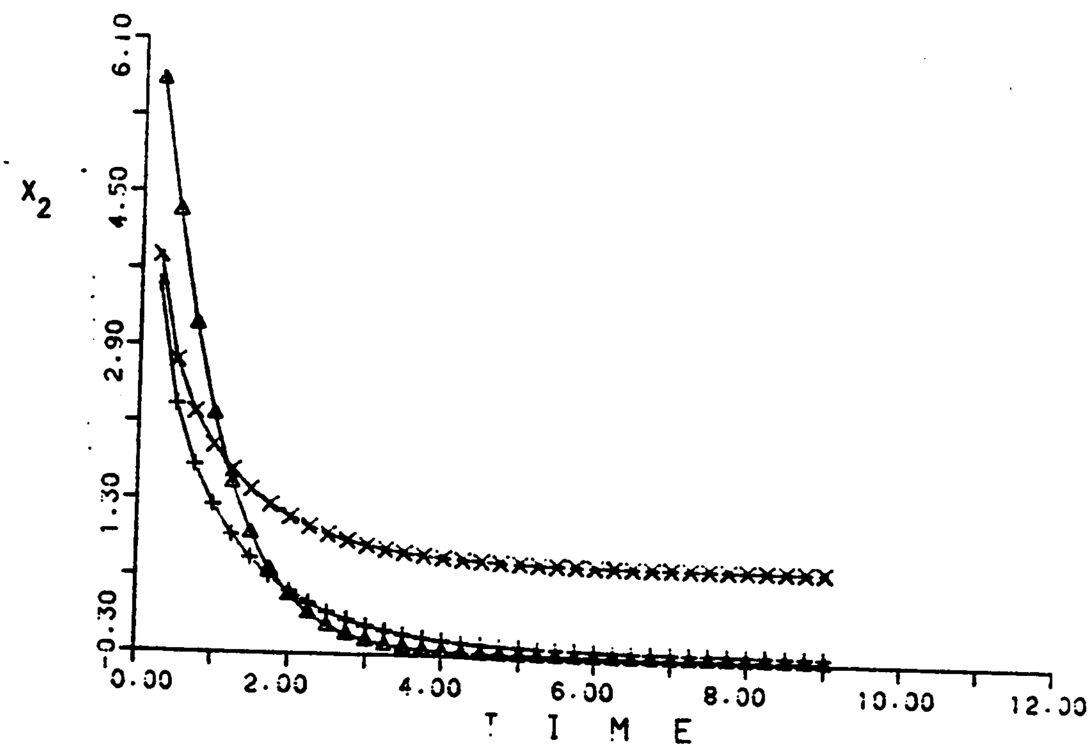

Figure 3.3(b). The state x_2 trajectory.

Δ optimal

+ developed model following

x model following as in [6,7]

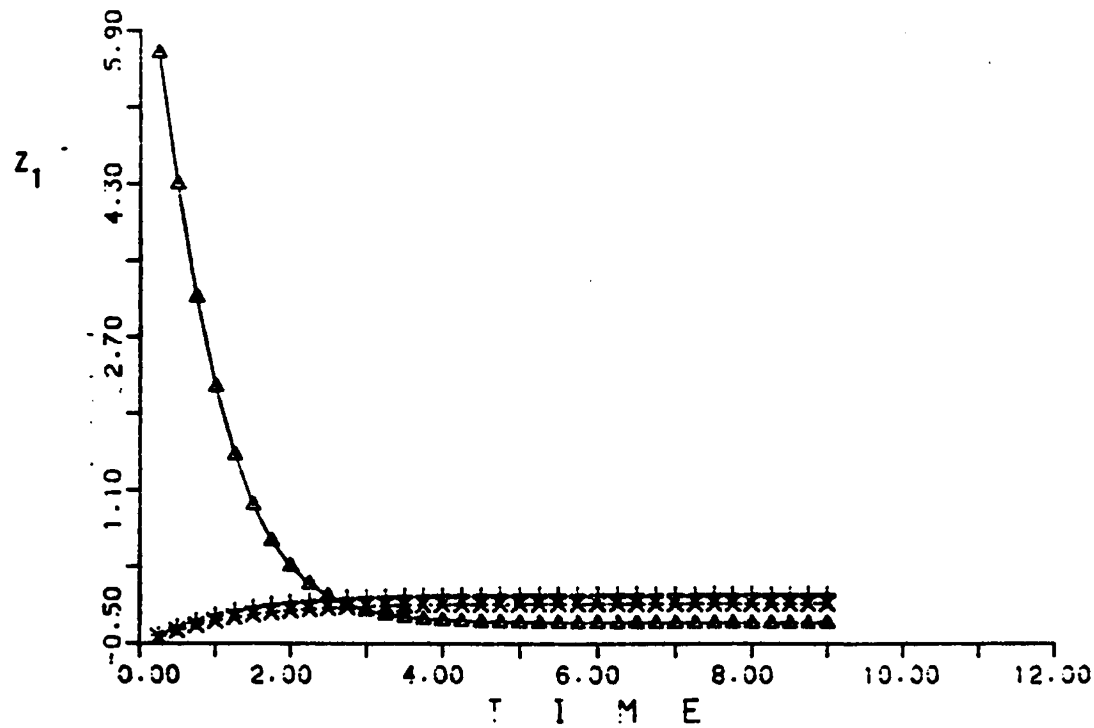

Figure 3.3(c). The interaction z_1 trajectory.

Δ optimal

+ developed model following.

x model following as in [6,7]

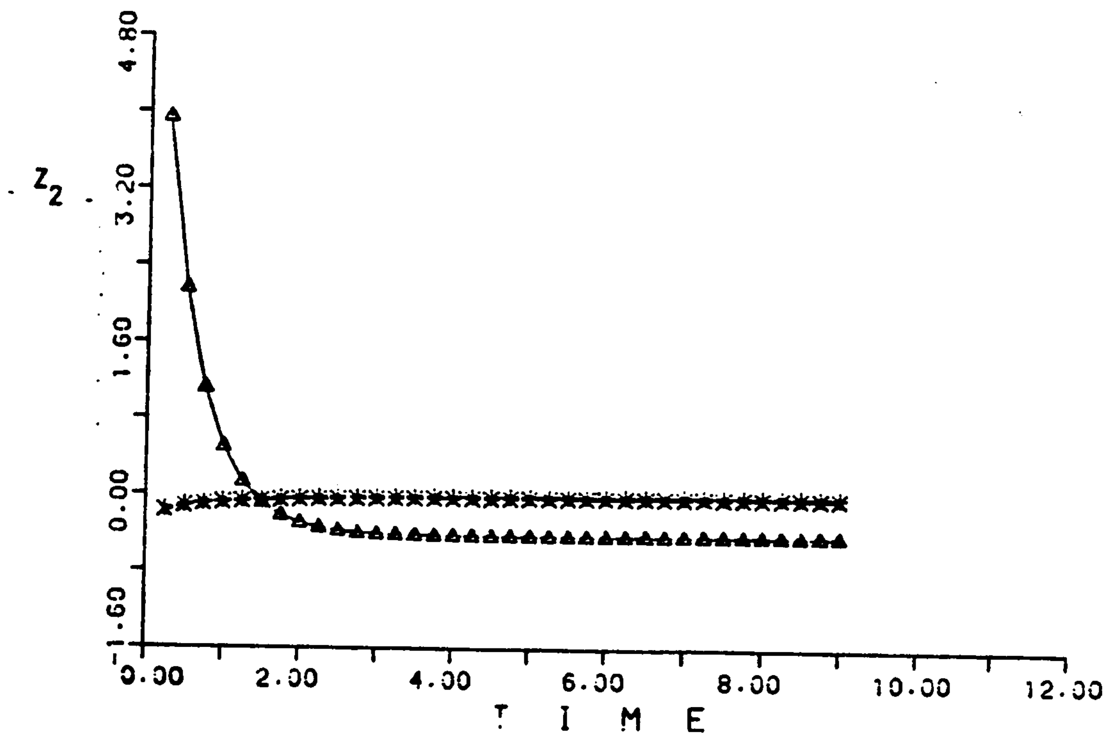

Figure 3.3(d). The interaction z_2 trajectory.

Δ optimal

+ developed model following.

x model following as in [6,7]

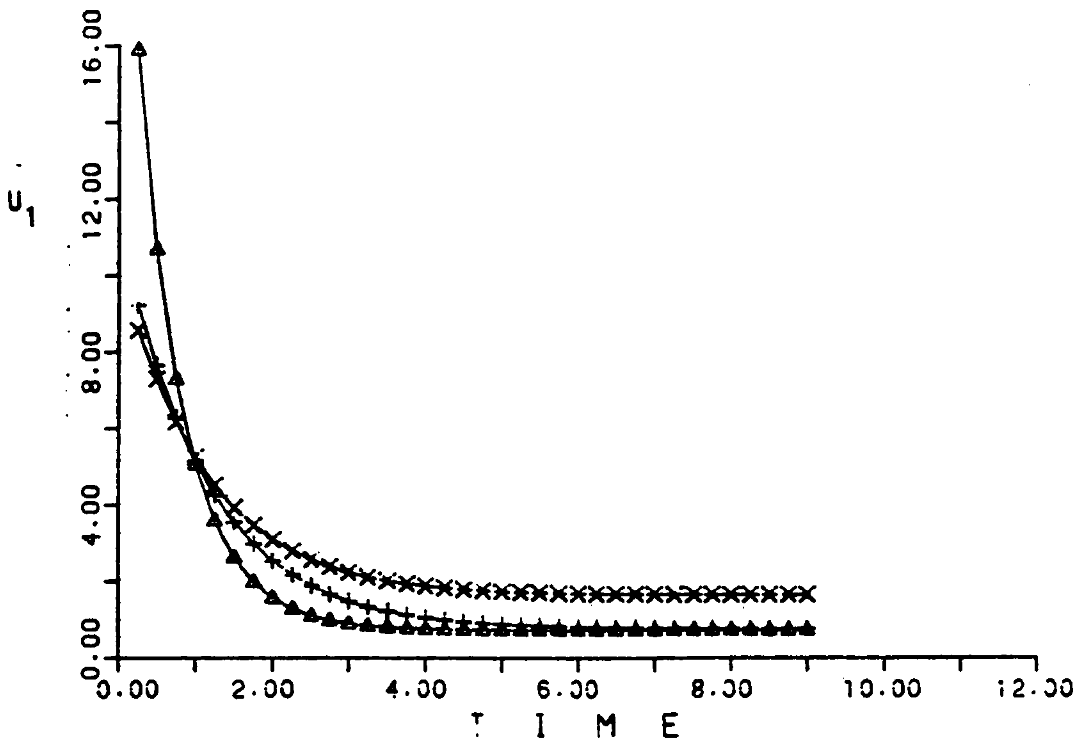

Figure 3.3(e). The control u_1 trajectory

Δ optimal

+ developed model following.

x model following as in [6,7]

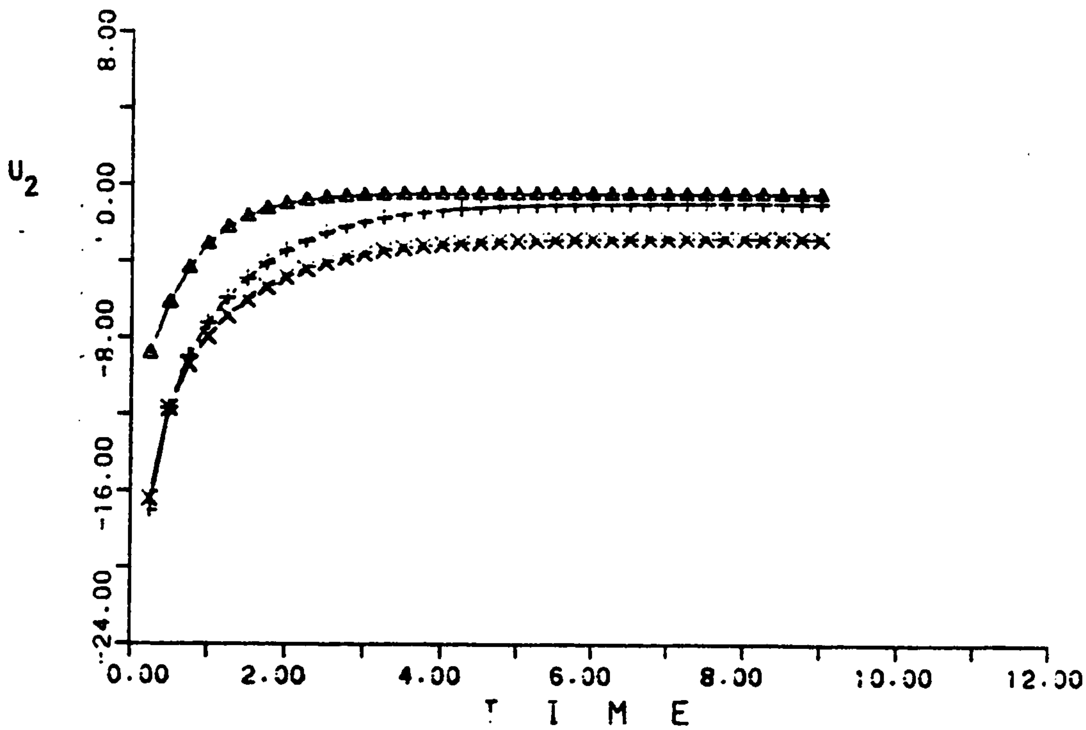

Figure 3.3(f). The control u_2 trajectory

Δ optimal

+ developed model following.

x model following as in [6,7]

As a comparison between the optimal control approach and the decentralized control using model following approach, we notice that the performance measure of the developed decentralized approach is less than the one proposed in [6,7] (Table I) as compared to the optimal value. The state trajectories are also closer to the optimal trajectories. There is a quite difference between the actual interaction and the one calculated by the model following technique in both approaches because the same model for interaction is used.

3.5 COMMENTS

(1) The decentralized controller in this approach has gains which are independent of the initial conditions and the control signal compensates a known input disturbance.

(2) The suboptimality in the controller design arises from the fact that:

a. The controller does not take into account all the states.

b. The gain matrix G_i depends on the crude interconnection model.

c. Although the output of the interconnection model is improved by the second optimization problem, there always remains a difference between z^* and z.

(3) Although the calculation is done in the subsystems level, there is a need to solve two optimization problems for each subsystem.

TABLE I. Comparison of the Performance Measure Between the Optimal Control Approach and the Model Following Approach Before and After Modification.

The approach	The performance measure
Optimal	142.20
Developed model following	211.43
Model following as in [6,7]	269.42

4. DECENTRALIZED CONTROL USING MODIFIED NTERACTION MODEL FOLLOWING APPROACH

4.1 INTRODUCTION

A modified interaction prediction approach is developed. The performance index is modified to include an extra term which accounts for the deviation of the interaction from the model. The interaction variables are treated as control signals.

An alternative approach to accommodate an input disturbance is also presented. An illustrative example is considered to show the potential of the developed algorithm. Simulation results are presented.

4.2 MODIFIED INTERACTION MODEL FOLLOWING APPROACH

In the approach proposed by Hassan and Singh [6,7] there is a need to solve two optimization problems. This leads to extensive computational effort especially in case of large scale systems. Also, in the process of deriving the algorithm, there are some approximations, e.g., in the first optimization problem, there is an approximation where the real interaction (z) is the same as the interaction generated by the model (z_m). Also, it is assumed that the same control signal is applied to both the subsystem model and the joriginal system model. The jimplementation of the controller necessitates a subsystem model and interaction model within the controller to generate the control signal.

Therefore, we modify the model following approach, to solve only one optimization problem without approximations.

4.2.1 Problem Formulation

We shall consider a dynamic system described by the Eqn. (2.1). The control input u(t) may be determined to minimize the performance measure given in Eqn. (2.2) where H = 0. Assuming that Q and R are diagonal matrices, the problem can be rewritten in the following decomposed form:

$$\min J = \sum_{i=1}^{N} \frac{1}{2} \int_{o}^{\infty} \{x_i^T Q_i x_i + u_i^T R_i u_i\} \, dt \qquad (4.1)$$

$$------i = 1,\ldots, N$$

Subject to:

$$\dot{x}_i = A_i x_i + B_i u_i + C_i z_i + E_i d_i \qquad (4.2)$$

$$z_i = \sum_{j=1}^{N} L_{ij} x_j$$

where x_i is an n_i-dimensional state vector of the i^{th} subsystem, u_i is an m_i-dimensional control vector, z_i is the q_i-dimensional interconnection vector and d_i is r_i-dimensional known disturbance vector.

z_{mi} is the required trajectory to be followed by the interaction z. It is represented by the following dynamical model:

$$\dot{z}_{mi} = A_{zi}\, z_{mi} + d_{zi} \tag{4.3}$$

A_{zi} is an arbitrary matrix, where (4.3) represents a crude model for the trajectory of z_i. The choice of A_{zi} was clarified in Chapter 3. d_{zi} is a constant input vector which is added to the crude model to be in a more general form.

Let us add a penalty term to the performance measure to penalize the performance of the system if interconnection variables do not follow z_{mi}. The performance measure will then take the form

$$\min J = \sum_{i=1}^{N} \frac{1}{2} \int_{0}^{\infty} (x_i^T Q_i x_i + u_i^T R_i u_i$$

$$+ (z_i - z_{mi})^T P_{zi} (z_i - z_{mi}))\, dt \tag{4.4}$$

where P_i is a positive definite matrix

Combining the vectors x_i and z_{mi} in one column vector y_i of dimension $(n_i + q_i)$, then Eqns. (4.2) - (4.4) take the form

$$J_i = \frac{1}{2}\int_0^{\infty} (y_i^T Q_{bi} y_i + u_i^T R_i u_i + z_i^T P_{zi} z_i - 2 y_i^T P_{bi} z_i)\, dt \quad (4.5)$$

Subject to:

$$\dot{y}_i = A_{bi} y_i + B_{bi} u_i + C_{bi} z_i + D_{bi} \quad (4.6)$$

where

$$A_{bi} = \begin{bmatrix} A_i & 0 \\ 0 & A_{zi} \end{bmatrix}, B_{bi} = \begin{bmatrix} B_i \\ 0 \end{bmatrix}$$

$$C_{bi} = \begin{bmatrix} C_i \\ 0 \end{bmatrix}, D_{bi} = \begin{bmatrix} B_i d_i \\ d_{zi} \end{bmatrix}, Q_{bi} = \begin{bmatrix} Q_i & 0 \\ 0 & P_{zi} \end{bmatrix}$$

$$P_{bi}^T = [0 \quad P_{zi}]$$

4.2.2 Development of the Decentralized Controller

The Hamiltonian for this problem is given by:

$$H_i = \frac{1}{2}\left(y_i^T Q_{bi} y_i + u_i^T R_i u_i + z_i^T P_{zi} z_i - 2 y_i^T P_{bi} z_i\right) +$$

$$\lambda_i^T (A_{bi} y_i + B_{bi} u_i + C_{bi} z_i + D_{bi}) \tag{4.7}$$

Then the necessary conditions for optimality are:

$$\dot{y}_i = A_{bi} y_i + B_{bi} u_i + C_{bi} z_i + D_{bi} \tag{4.8}$$

$$\dot{\lambda}_i = -Q_{bi} y_i + P_{bi} z_i - A_{bi}^T \lambda_i \tag{4.9}$$

$$u_i = -R_i^{-1} B_{bi} \lambda_i \tag{4.10}$$

$$z_i = -P_{zi}^{-1} C_{bi}^T \lambda_i + P_{zi}^{-1} P_{bi}^T y_i \tag{4.11}$$

Let us assume that $\lambda_i = K_i y_i + S_i$ (4.12)

Differentiating both sides of Eqn. (4.12) with respect to t, and substituting for λ_i and x_i. This results into the following matrix differential equations

$$\dot{K}_i = -K_i A_{bi} + K_i B_{bi} R_i^{-1} B_{bi}^T K_i + K_i C_{bi} P_{zi}^{-1} C_{bi}^T K_i$$

$$- K_i C_{bi} P_{zi}^{-1} P_{bi}^T - Q_{bi} - P_{bi} P_{zi}^{-1} C_{bi}^T K_i$$

$$+ P_{bi} P_{zi}^{-1} P_{bi}^T - A_{bi}^T K_i \tag{4.13}$$

$$\dot{S}_i = K_i B_{bi} R_i^{-1} B_{bi}^T S_i + K_i C_{bi} P_{zi}^{-1} C_{bi}^T S_i - P_{bi} P_{zi}^{-1} C_{bi} S_i$$

$$- A_{bi}^T S_i + K_i D_i \tag{4.14}$$

Define:

$$A_{ai} = A_{bi} + C_{bi} P_{zi}^{-1} P_{bi}^T \tag{4.15}$$

$$B_{ai} = [B_{bi} : C_{bi}] \tag{4.16}$$

$$Q_{ai} = Q_{bi} - P_{bi} P_{zi}^{-1} P_{bi}^T \tag{4.17}$$

$$R_{ai} = \begin{bmatrix} R_i & 0 \\ & \\ 0 & P_{zi} \end{bmatrix} \tag{4.18}$$

Substituting for A_{bi}, B_{bi}, C_{bi}, P_{bi} and Q_{bi} in Eqns. (4.15) - (4.18), we get

$$A_{ai} = \begin{bmatrix} A_i & C_i \\ & \\ 0 & A_{zi} \end{bmatrix} \tag{4.19}$$

$$B_{ai} = \begin{bmatrix} B_i & C_i \\ & \\ 0 & 0 \end{bmatrix} \tag{4.20}$$

$$Q_{ai} = \begin{bmatrix} Q_i & 0 \\ & \\ 0 & 0 \end{bmatrix} \tag{4.21}$$

$$R_{ai} = \begin{bmatrix} R_i & 0 \\ & \\ 0 & P_{zi} \end{bmatrix} \tag{4.22}$$

Then the Eqns. (4.13) and (4.14) in the standard forms:

$$\dot{K}_i = -K_i\, A_{ai} - A_{ai}^T\, K_i + K_i\, B_{ai}\, R_{ai}^{-1}\, B_{ai}^T\, K_i - Q_{ai} \tag{4.23}$$

$$\dot{S}_i = -\,[A_{ai} - B_{ai}\, R_{ai}^{-1}\, B_{ai}^T\, K_i]^T\, S_i + K_i\, D_i \tag{4.24}$$

Equation (4.23) is the standard matrix Riccati equation.

To have the steady state values for K and S, let $\dot{K} = \dot{S} = 0$ and solve the resulting algebraic equations form of (4.23) and (4.24).

Substituting for λ_i in the control law u_i and in the interaction z_i, we will get:

$$u_i^* = - R_i^{-1} B_i K_i y_i + v_{ui} \tag{4.25}$$

$$z_i^* = - P_{zi}^{-1} [C_i^T K_i - P_{bi}^T] y_i + v_{zi} \tag{4.26}$$

u_i^* and z_i^* can be written in terms of u_i and z_{mi} as kfollows:

$$u_i^* = -G_{ix} x_i - G_{iz} z_{mi} + v_{ui} \tag{4.27}$$

$$z_i^* = -K_{ix} x_i - K_{iz} z_{mi} + v_{zi} \tag{4.28}$$

The resulting gain matrices will depend on the chosen A_{zi} matrix and on the weighting matrix P_{zi}. From Eqn. (4.27), we can see that the control does not depend on z_i (i.e. the actual interaction) as in the previous method, but it depends on z_{mi} generated by the model.

Since, the gain matrix is a function of the crude model, therefore a good choice for the crude model will yield better results. Another improvement can be achieved by using z_i^* for generating the control signal.

For simulation purpose we can calculate z_i using Eqn. (4.28) and this is referred to as offline. While the simulation of the whole system with the actual interaction is referred to as online.

Thus the dynamical equations for the subsystem have forms:

<u>**Offline Form:**</u>

$$\dot{x}_i = A_i x_i + B_i u_i^* + C_i z_i^* + E_i d_i \qquad (4.29)$$

$$\dot{z}_{mi} = A_{zi} z_{mi} + d_{zi} \qquad (4.30)$$

$$z_i^* = -K_{ix} x_i - K_{iz} z_{mi} + v_{zi}' \qquad (4.31)$$

$$u_i^* = -G_{ix} x_i - G_{iz} z_{mi} + v_{ui}' \qquad (4.32)$$

This form is depicted in the block diagram in Fig. 4.1.

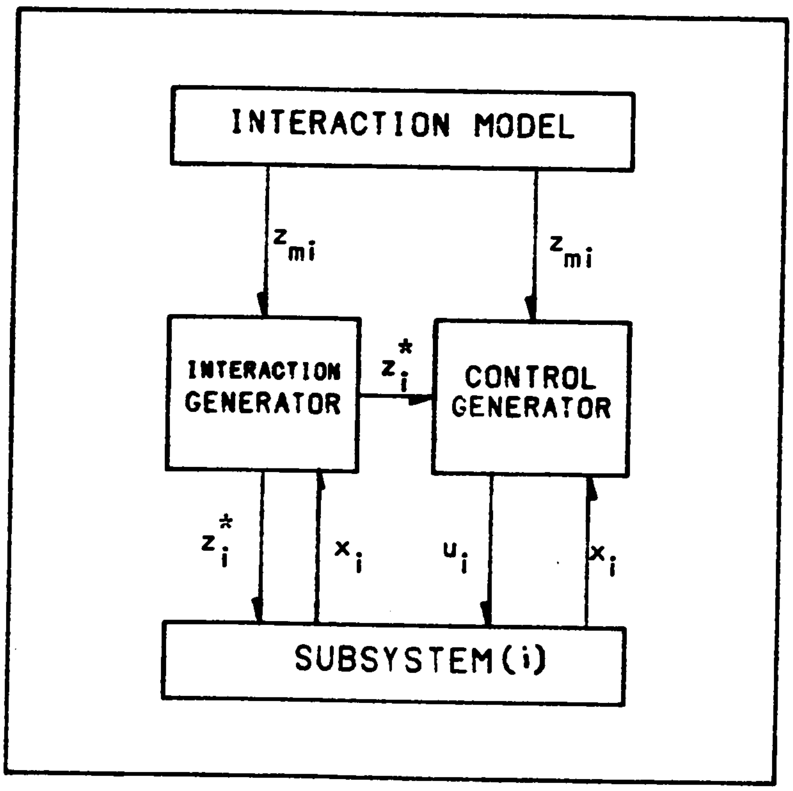

Figure 4.1. Block diagram of modified decentralized controller (off-line).

<u>Online Form:</u>

$$\dot{x}_i = A_i x_i + B_i u_i^* + C_i z_i + E_i d_i \tag{4.33}$$

$$\dot{z}_{mi} = A_{zi} z_{mi} + d_{zi} \tag{4.34}$$

$$z_i = \sum_{j=1}^{N} L_{ij} x_j \tag{4.35}$$

$$z_i^* = -K_{ix} x_i - K_{iz} z_{mi} + v_{zi} \tag{4.36}$$

$$u_i^* = -G_{ix} x_i - G_{iz} z_{mi} + v_{ui} \tag{4.37}$$

or

$$u_i^* = -G_{ix} x_i - G_{iz} z_i^* + v_{ui} \tag{4.38}$$

This form is depicted in the block diagram in Fig. 4.2.

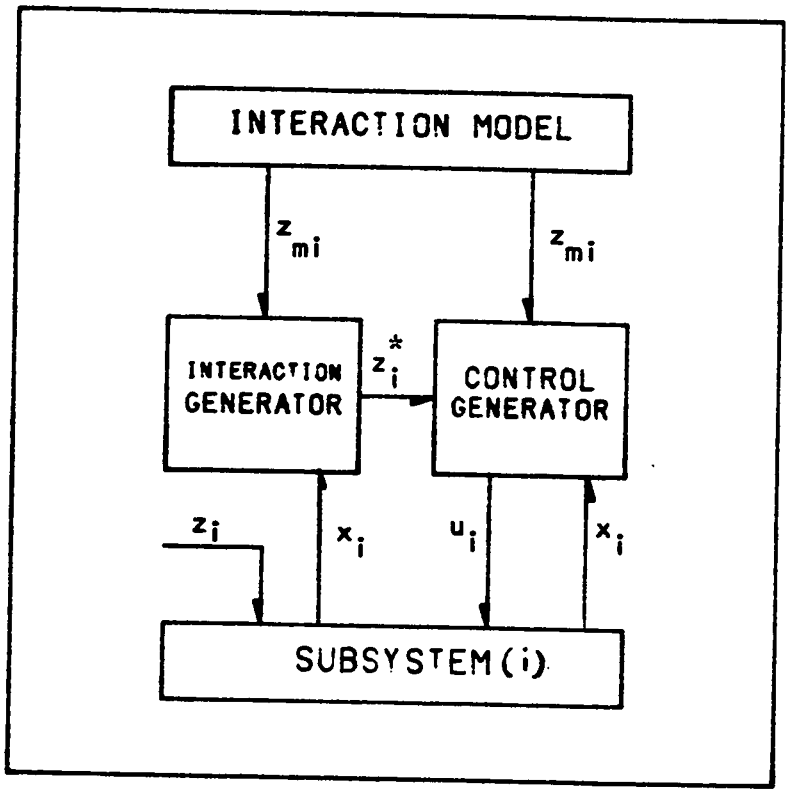

Figure 4.2. Block diagram of a modified decentralized controller (on-line).

Alternative Approach:

The decentralized controller presented in Section (4.2.2) can be also developed as follows:

Define $e_i = z_i - z_{mi}$ (4.39)

as the error necessary to update the interaction trajectory as generated from the crude model.

From (4.39) we can get:

$$z_i = z_{mi} + e_i \tag{4.40}$$

We can rewrite the state equation in the following form:

$$\dot{x}_i = A_i x_i + B_i u_i + C_i (z_{mi} + e_i) + E_i d_i \tag{4.41}$$

$$\dot{z}_{mi} = A_{zi} z_{mi} + d_{zi} \tag{4.42}$$

Equations (4.4), (4.41) and (4.42) can be combined together to form a modified optimization problem in the following form:

$$\min J = \sum_{i=1}^{N} \frac{1}{2} \int_{o}^{\infty} (y_i^T Q_{ai} y_i + w_i^T R_{ai} w_i)\, dt \tag{4.43}$$

Subject to:

$$\dot{y}_i = A_{ai} y_i + B_{ai} w_i + D_{ai} \tag{4.44}$$

where

$$y_i = \begin{pmatrix} x_i \\ \\ z_{mi} \end{pmatrix} \quad , w_i = \begin{pmatrix} u_i \\ \\ e_i \end{pmatrix}$$

$$A_{ai} = \begin{pmatrix} A_i & C_i \\ \\ 0 & A_{zi} \end{pmatrix} \quad ,B_{ai} = \begin{pmatrix} B_i & C_i \\ \\ 0 & 0 \end{pmatrix}$$

$$Q_{ai} = \begin{pmatrix} Q_i & 0 \\ \\ 0 & 0 \end{pmatrix} \quad , R_{ai} = \begin{pmatrix} R_i & 0 \\ \\ 0 & P_{zi} \end{pmatrix}$$

$$D_i = \begin{pmatrix} E_i d_i \\ \\ d_{zi} \end{pmatrix}$$

After formulating the problem in the standard form, the optimal control input w_i vector can be determined. w_i vector will consist of u_i and e_i. The resulting gain matrix depends on the chosen matrix A_{zi} and on the weighting matrix P_{zi}.

Then the solution of the Eqns. (4.43) and (4.44) will be:

$$w_i^* = -R_{ai}^{-1} B_{ai} P_i y_i + v_i \tag{4.45}$$

where P_i is the solution of steady state Riccati Eqn. (2.4) and v_i is the solution of the constant term Eqn. (2.5).

The optimal control can be rewritten as:

$$u_i^* = -G_{ix} x_i - G_{iz} z_{mi} + v_{ui} \tag{4.46}$$

$$e_i^* = -K_{ix} x_i - K_{iz} z_{mi} + v_{ei} \tag{4.47}$$

From the above equation, we can see that the control does not depend on z_i (i.e. the actual interaction) as in the previous method, but it can use z_{mi} which is generated by the model.

Also, it is obvious that the gain matrix is a function of the crude model, therefore, a good choice for the crude model will give better results.

Another improvement can be achieved by having $z_i^* = z_{mi} + e_e^*$ to generate the control.

Thus the dynamical equations for the subsystem will be:

$$\dot{x}_i = A_i x_i + B_i u_i + C_i z_i + E_i d_i \tag{4.48}$$

$$\dot{z}_{mi} = A_{zi} z_{mi} + d_{zi} \tag{4.49}$$

$$e_i^* = -K_{ix} x_i - K'_{iz} z_{mi} + v_{ei} \tag{4.50}$$

$$u_i^* = -G_{ix} x_i - G_{iz} (z_{mi} + e_i^*) + v_{ui} \tag{4.51}$$

The Eqn. (4.51) can be rewritten to be

$$u_i^* = -G_{ix}^* x_i - G_{iz}^* z_{mi} + v_{ui}^* \tag{4.52}$$

4.3 ILLUSTRATIVE EXAMPLE

Let us consider the following problem [17]. A dynamical system is described by the differential equations:

$$\dot{x}_1(t) = x_2(t) - u_1(t)$$
$$\dot{x}_2(t) = 2\, x_1(t) - x_2(t) + u_2(t)$$

It is required to determine the optimal control function u(tr) for the above system while minimizing the performance measure,

$$J = \int_0^\infty (x_1^2(t) + 0.5\, x_2^2(t) + 0.5\, u_1^2(t) + 0.5\, u_2(t))\, dt$$

Let us rewrite these equations in the standard form:

$$\dot{x} = A\,x + B\,u + E\,d$$

where

$$A = \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \quad E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$d = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

$$J = \frac{1}{2} \int_0^\infty (x^T Q\, x + u^T R\, u)\, dt$$

where

$$Q = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}, \quad R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

<u>System Decomposition:</u>

The system is decomposed into two subsystems. The first subsystem is represented by the state x_1:

$$\dot{x}_1 = z_1 - u_1$$
$$z_1 = x_2$$

and the second subsystem is represented by the state x_2:

$$\dot{x}_2 = -x_2 + 2z_2 + u_2$$
$$z_2 = x_1$$

The performance measure is also decomposed into two parts:

$$J_1 = \int_o^\infty (x_1^2 + 0.5\, u_1^2)\, dt$$

and

$$J_2 \int_o^\infty (0.5\, x_2^2 + 0.5\, u_2^2)\, dt$$

Now, we have two subproblems. Let us choose a crude model for the interaction for each subsystem of the form:

$$\dot{z}_1 = -z_1$$
$$\dot{z}_2 = -2z_2$$

Optimal Controller:

The optimal control takes the form

$$u = -R^{-1} B^T P x$$

where the Riccati matrix

$$P = \begin{bmatrix} 2.2042 & 0.9316 \\ 0.9316 & 0.7307 \end{bmatrix}$$

Then,

$$u_1 = +2.2042 \quad x_1 + 0.9316\, x_2$$
$$u_2 = -0.9316 \quad x_1 - 017307\, x_2$$

Simulation results showing states trajectories, interaction variables and the control functions are given in Fig. 4.3.

Decentralized Model-Following Controller:

Formulating the two subproblems in the model-following form, the first subproblem matrices are:

$$A_1 = \begin{bmatrix} 0.0 & 1.0 \\ 0.0 & -1.0 \end{bmatrix}, B_1 = \begin{bmatrix} -1.0 \\ 0.0 \end{bmatrix}, D_1 = \begin{bmatrix} 0.0 \\ 0.0 \end{bmatrix}$$

$$Q_1 = \begin{bmatrix} 2.0 & 0.0 \\ 0.0 & 0.0 \end{bmatrix}, R_1 = 1.0$$

and the second matrices are:

$$A_2 = \begin{bmatrix} -1.0 & 2.0 \\ 0.0 & -2.0 \end{bmatrix}, B_2 = \begin{bmatrix} 1.0 \\ 0.0 \end{bmatrix}, D_2 = \begin{bmatrix} 0.0 \\ 0.0 \end{bmatrix}$$

$$Q_2 = \begin{bmatrix} 1.0 & 0.0 \\ 0.0 & 0.0 \end{bmatrix}, R_2 = 1.0$$

Gain matrices for each subsystem are computed as:

$G_1 = [-1.4142 \quad , \quad -0.5858]$

$K_1 = 0.0690$

$G_2 = [0.4142 \quad , \quad 0.2426]$

$K_2 = 0.1296$

The equations required to implement the controller of the first subsystem are:

$$\dot{z}_{m1} = -z_{m1}$$
$$\dot{x}_{m1} = -1.4742\, x_{m1} + 0.4142\, z_{m1}$$
$$x_{e1} = x_1 - x_{m1}$$
$$z_{e1} = -0.069\, x_{e1}$$
$$z_1 = z_{m1} + z_{e1}$$
$$u_1 = 1.4142\, x_1 + 0.5858\, z_1$$

The first controller will have x_1 as input and u_1 as output.

The equations required to implement the controller of the second subsystem are:

$$\dot{z}_{m2} = -2\, z_{m2}$$
$$\dot{x}_{m2} = -1.4142\, x_{m2} + 1.7574\, z_{m2}$$
$$x_{e2} = x_2 - x_{m2}$$
$$z_{e2} = 0.1296\, x_{e2}$$
$$z_2 = z_{m2} + z_{e2}$$
$$u_2 = -0.4142\, x_2 - 0.2426\, z_2$$

The second controller will have x_2 as input and u_2 as output.

Simulation results showing the state trajectories, interaction variables and the control functions are given in Fig. 4.3.

Modified Decentralized Model-Following Controller:

Putting the two subproblems in the modified model-following form, the first

$$A_1 = \begin{bmatrix} 0.0 & 1.0 \\ 0.0 & -1.0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -1.0 & 1.0 \\ 0.0 & 0.0 \end{bmatrix}$$

$$D_1 = \begin{bmatrix} 0.0 \\ 0.0 \end{bmatrix}, \quad Q_1 = \begin{bmatrix} 2.0 & 0.0 \\ 0.0 & 0.0 \end{bmatrix}, \quad R_1 = \begin{bmatrix} 1.0 & 0.0 \\ 0.0 & 1.0 \end{bmatrix}$$

and the second subproblem matrices are:

$$A_2 = \begin{bmatrix} -1.0 & 2.0 \\ 0.0 & -2.0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1.0 & 2.0 \\ 0.0 & 0.0 \end{bmatrix}$$

$$D_2 = \begin{bmatrix} 0.0 \\ 0.0 \end{bmatrix}, \quad Q_2 = \begin{bmatrix} 1.0 & 0.0 \\ 0.0 & 0.0 \end{bmatrix}, \quad R_2 = \begin{bmatrix} 1.0 & 0.0 \\ 0.0 & 0.0 \end{bmatrix}$$

Gain matrices for each subsystem are computed as:

G_1 = [-1.000 , -0.333]

K_1 = [1.000 , -0.667]

G_2 = [0.2899 , 0.1303

K_2 = [0.5798 , -0.7394]

The equations required to implement the controller of the first subsystem are:

$$\dot{z}_{m1} = -z_{m1}$$

$$z_1 = -x_1 + 0.667\, z_{m1}$$

$$u_1 = x_1 + 0.333\, z_1$$

The equations required to implement the controller of the second subsystem are:

$$\dot{z}_{m2} = -2\, z_{m2}$$

$$z_2 = -0.5798\, x_2 + 0.7394\, z_{m2}$$

$$u_2 = -0.2899\, x_2 - 0.1303\, z_2$$

Simulation results showing the state trajectories, interaction variables and the control functions in off-line case and on-line case are given in Fig. 4.3.

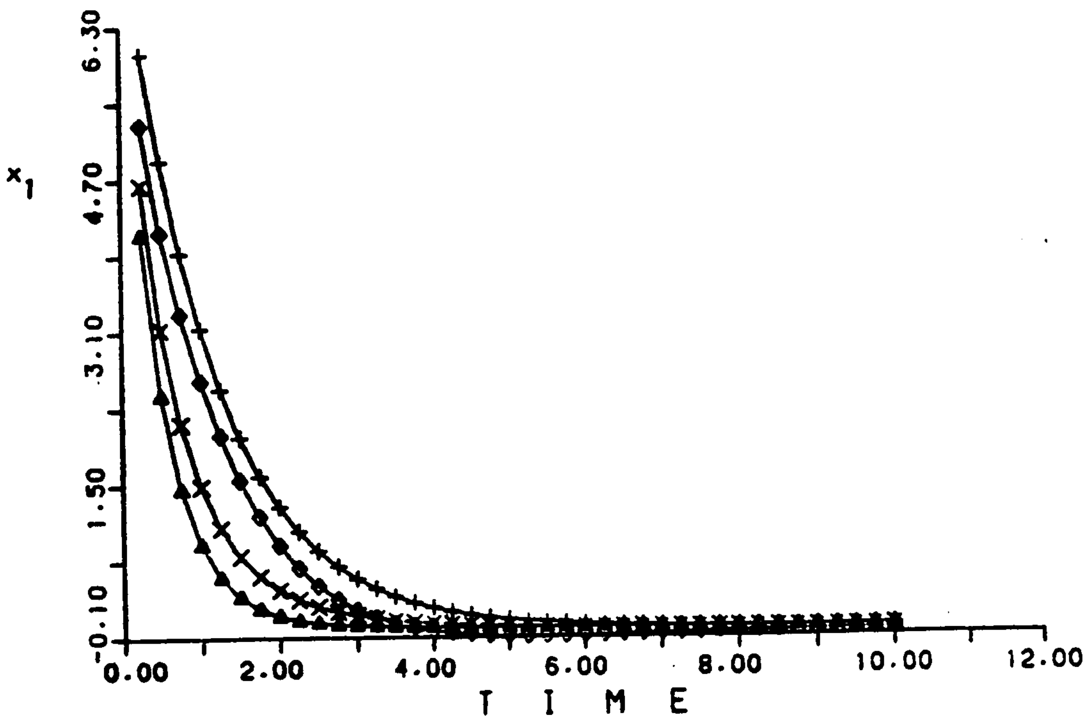

Figure 4.3(a). The state x_1 trajectory.

Δ	optimal
+	model following
x	modified model following (off-line)
◊	modified model following (on-line)

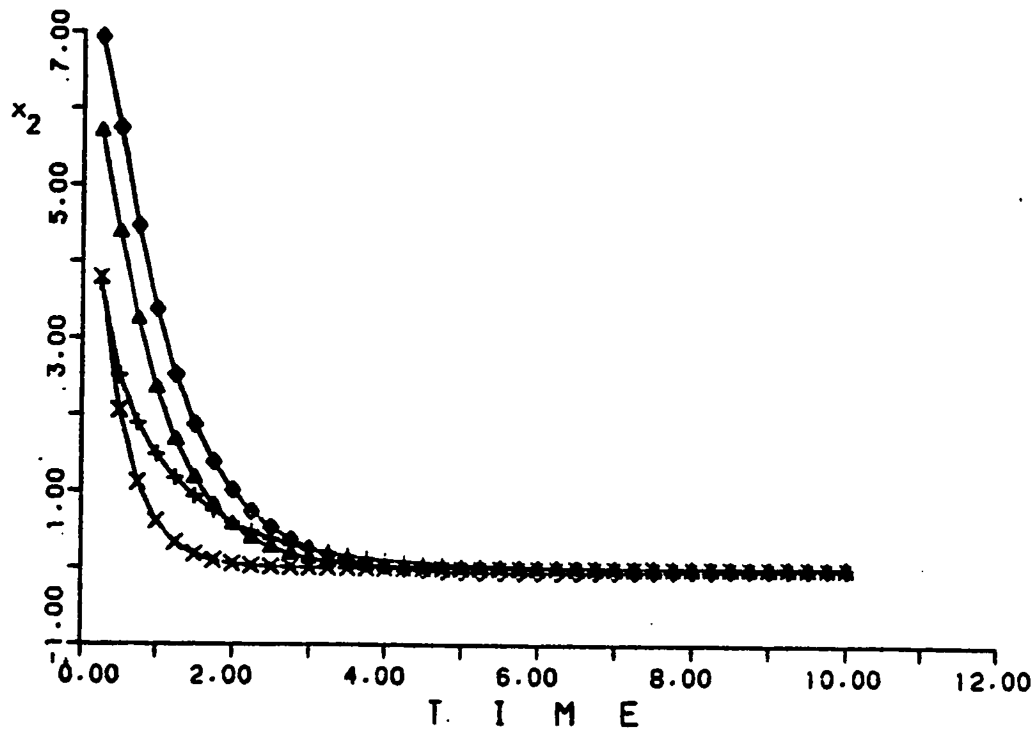

Figure 4.3(b). The state x_2 trajectory.

Δ	optimal
+	model following
x	modified model following (off-line)
◊	modified model following (on-line)

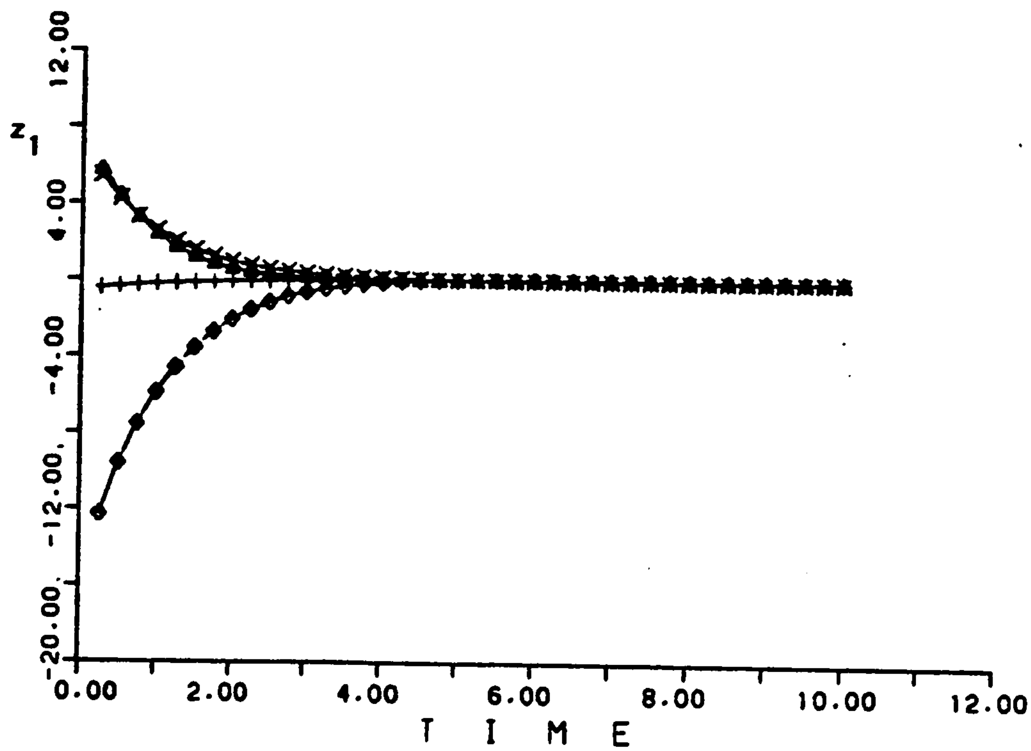

Figure 4.3(c). The interaction z_1 trajectory.

Δ	optimal
+	model following
x	modified model following (off-line)
◊	modified model following (on-line)

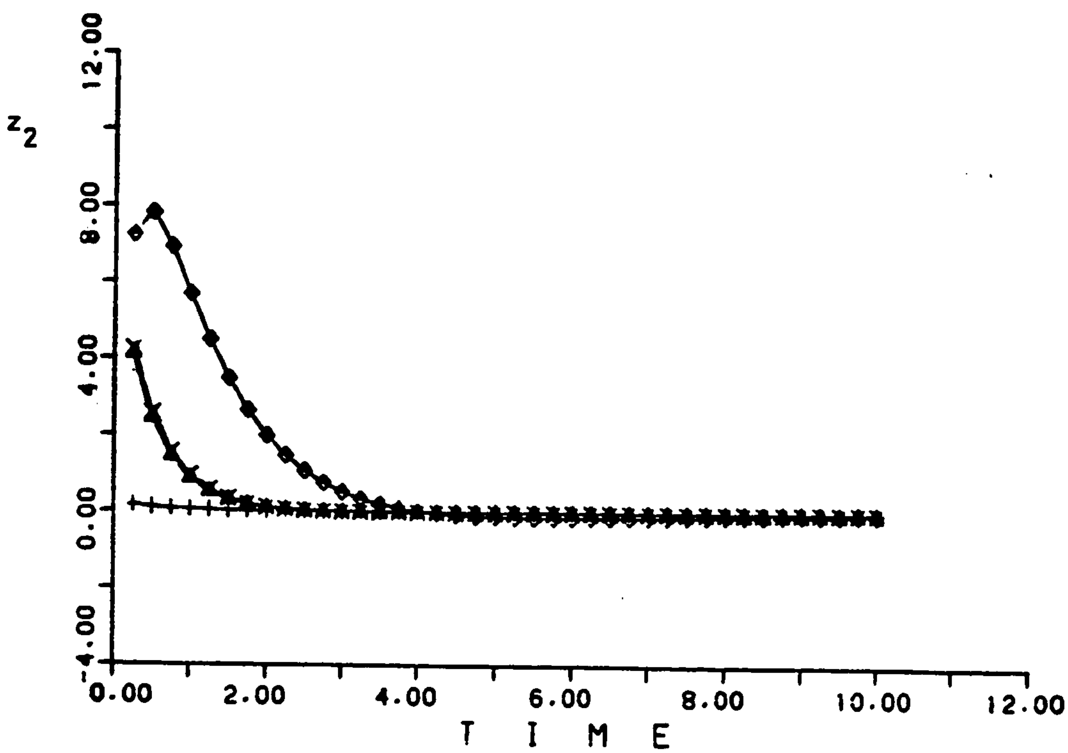

Figure 4.3(d). The interaction z_2 trajectory.

Δ	optimal
+	model following
x	modified model following (off-line)
◊	modified model following (on-line)

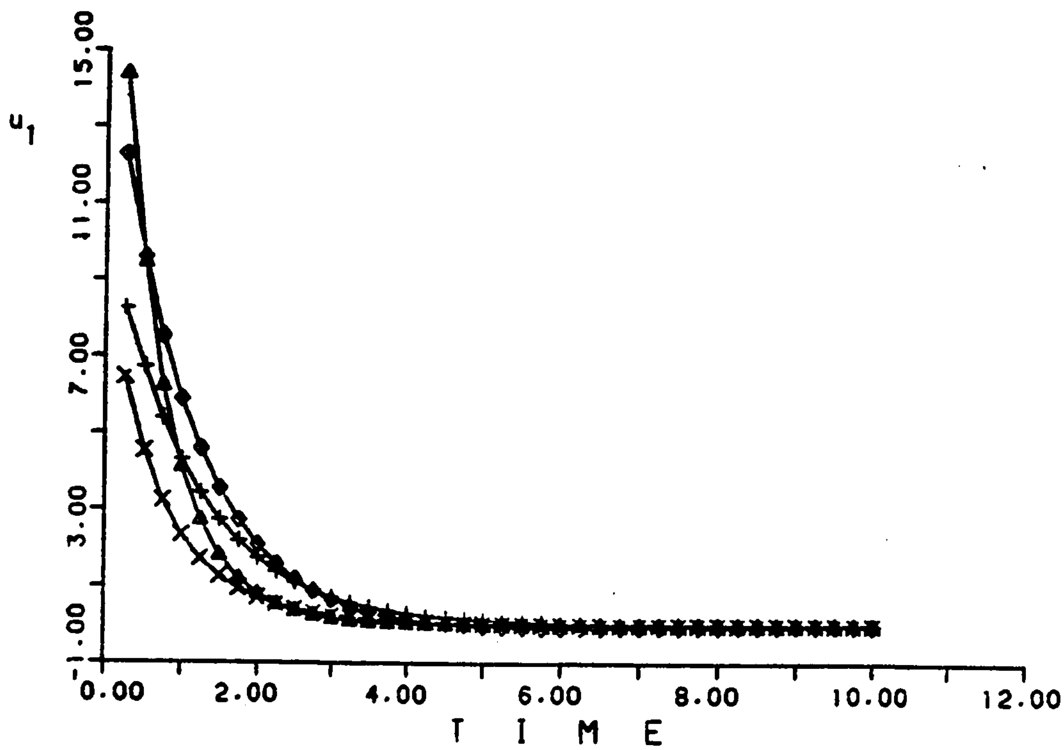

Figure 4.3(e). The control u_1 trajectory.

△ optimal

+ model following

x modified model following (off-line)

◊ modified model following (on-line)

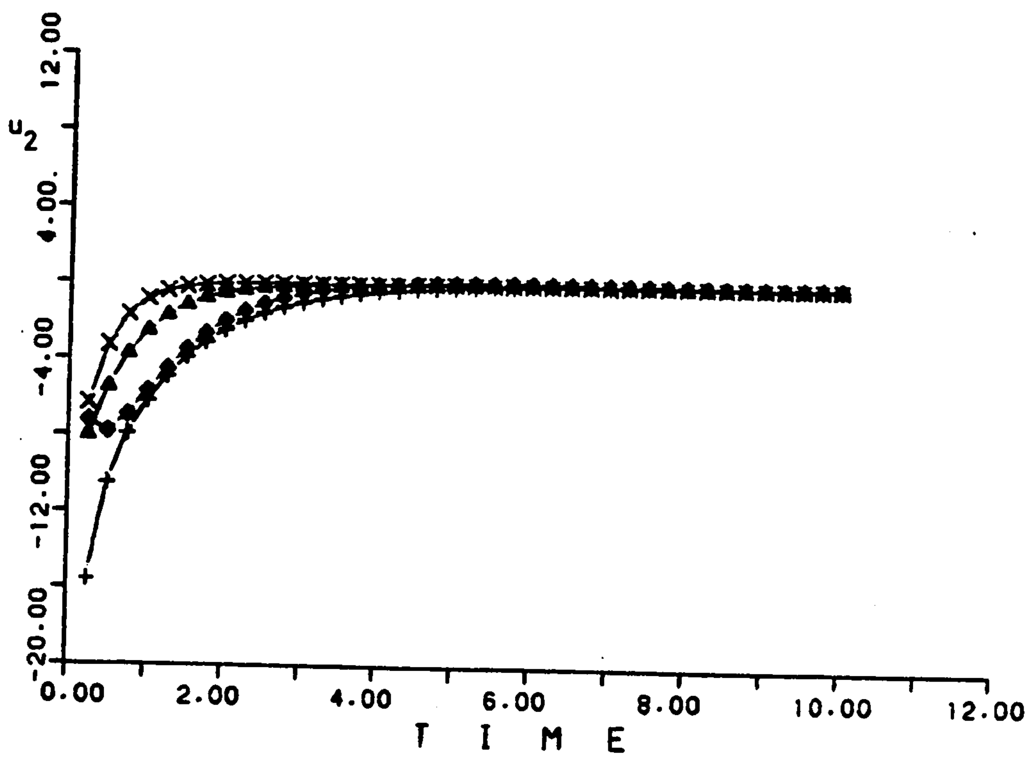

Figure 4.3(e). The control u_2 trajectory.

Δ	optimal
+	model following
x	modified model following (off-line)
◊	modified model following (on-line)

Comparison:

Since there is a strong similarity between the actual interaction and the one calculated in the offline case of the modified model-following approach. The offline case can be used to generate the interaction in the online case. In the modified model-following approach, we achieved the minimum control effort in both cases. Also, the calculated performance measure, as shown in Table II, using the modified model following approach is better than the performance measure achieved by the model-following approach of Singh et al [6,7]. The interaction in both the modified model-following and the normal model-following depends strongly on the crude model. Therefore it can be improved by choosing better crude model.

TABLE II. Table Showing the Performance Measure Using Different Approaches.

The approach	The performance measure
Optimal	117.07
Model-following	172.06
Modified Model-following (Online)	135.64

4.4 COMMENTS

General comments regarding the developed modified interaction approach can be drawn as follows:

(1) There is no need to have a model for the subsystem in the controller.

(2) The gain matrices can be calculated by solving only one optimization problem for each subsystem.

(3) The resulting solution of the problem is suboptimal due to:

a. The calculation is done on the subsystem level.

b. The gain matrices depend on the interaction model (A_{zi}) and the weighting matrix (P_{zi}).

c. There always remains a difference between z^* and z.

4.5 DECENTRALIZED CONTROLLER WITH THE IDEA OF DISTURBANCE REJECTION [8,9]

The idea of having a nonlinear function of the control and its derivatives in the performance mesaure as used in disturbance rejection technique proposed by Johnson [24] is implemented in this approach. Let us consider u_i to comprise two components, i.e.

$$u_i = u_{1i} + u_{2i} \tag{4.53}$$

where u_{1i} is the local optimal control when the interactions between the subsystems are ignored, and u_{2i} is an additional control for i^{th} subsystem that will be generated using the idea of the disturbance rejection by considering the jinteractions as unknown disturbances. Then a performance measure of the form

$$J = \frac{1}{2} \sum_{i=1}^{N} \int_{o}^{\infty} [x_{is}^{T} Q_i x_{is} + \Phi_i (u_{2i}, \dot{u}_{2i})] \, dt \tag{4.54}$$

will be minimized.

Where x_{is} is the suboptimal trajectories resulting from the application of the local optimal control u_{1i}.

4.5.1 Development of the Controller

We shall consider the optimization of a subproblem described by Eqns. (4.1), (4.2).

By ignoring the interaction variables z_i, Eqn. (4.2) can be written in the following form

$$\dot{x}_{is} = A_i x_{is} + B_i u_{1i} + E_i d_i \qquad (4.55)$$

Solving this subproblem, we can get the first component of the control u_i as follow:

$$u_{1i} = -R_i^{-1} B_i^T P_i x_{is} - R_i^{-1} B_i^T s_i \qquad (4.56)$$

where P_i is a solution of a Riccati Eqn. (2.4) and s_i is the solution of the matrix differential Eqn. (2.5).

Substituting for u_{1i} in Eqn. (4.2) we will get:

$$\dot{x}_{is} = A_i^* x_{is} + C_i z_i + D_i \qquad (4.57)$$

where

$$A_i^* = A_i - B_i R_i^{-1} B_i^T P_i \qquad (4.58)$$

$$D_i = d_i - B_i R_i^{-1} B_i^T s_i \qquad (4.59)$$

Let assume a crude model for the interaction given by Eqn. (4.3) and from Eqn. (4.39) we get

$$z_{mi} = z_i - e_i \tag{4.60}$$

Substituting for z_{mi} from Eqn. (4.60) into Eqn. (4.3) we get:

$$\dot{z}_i = A_{zi}\, z_i + \dot{e}_i - A_{zi}\, e_i + d_{zi} \tag{4.61}$$

To compensate the error (e_i) between z_i and z_{mi} we will use u_{2i} such that

$$B_i\, u_{2i} - C_i e_i = 0 \tag{4.62}$$

Hence,

$$e_i = E_i\, u_{2i} \tag{4.63}$$

where $\quad E_i = (C_i^T\, C_i)^{-1}\, C_i^T\, B_i$,

Substituting for e_i from Eqn. (4.03) into Eqn. (4.61), we get:

$$\dot{z}_i = A_{zi}\, z_i + E_i\, \dot{u}_{2i} - A_{zi}\, E_i\, u_{2i} + d_{zi} \tag{4.64}$$

Let us write

$$v_i = E_i \dot{u}_{2i} - A_{zi} E_i u_{2i} \tag{4.65}$$

Let us define the function

$$\Phi_i(u_{2i}, \dot{u}_{2i}) = v_i^T R_i v_i \tag{4.66}$$

By combining (4.57), (4.64), (4.65), (4.66) and (4.54), the optimal decentralized control problem can be rewritten as

$$\min J = \sum_{i=1}^{N} \int_{o}^{\infty} (y_i^T Q_{ai} y_i + v_i^T R_i v_i)\, dt \tag{4.67}$$

subject to

$$\dot{y}_i = A_{ai} y_i + B_{ai} v_i + D_{ai} \tag{4.68}$$

where

$$y_i = \begin{bmatrix} x_i \\ \\ z_i \end{bmatrix} \quad , Q_{ai} = \begin{bmatrix} Q_i & 0 \\ & \\ 0 & 0 \end{bmatrix}$$

$$A_{ai} = \begin{bmatrix} A_i^* & C_i \\ 0 & A_{zi} \end{bmatrix}, \quad B_{ai} = \begin{bmatrix} 0 \\ I \end{bmatrix}, \quad D_{ai} = \begin{bmatrix} D_i \\ d_{zi} \end{bmatrix}$$

The solution of this problem can be written as

$$v_i = -G_{ix} x_i - G_{iz} z_i + \eta_i \tag{4.69}$$

Substituting for v_i from Eqn. (4.69) into Eqn. (4.65), we get

$$\dot{u}_{2i} = A_{ui} u_{2i} - H_i x_i - F_i z_i + S_i \tag{4.70}$$

where

$$A_{ui} = (E_i^T E_i)^{-1} E_i^T A_{zi} E_i$$

$$H_i = (E_i^T E_i)^{-1} E_i^T G_{ix}$$

$$F_i = (E_i^T E_i)^{-1} E_i^T G_{iz}$$

$$S_i = (E_i^T E_i)^{-1} E_i^T \eta_i$$

5. THE DECENTRALIZED CONTROL SYSTEM OF HOT ROLLING FINISHING MILL

5.1 INTRODUCTION

In this chapter, we will apply the developed approaches in the design of the decentralized control system of the hot rolling finishing mill as an application. The results of the simulation of the developed approaches will be compared with the optimal case.

5.2 GENERAL DESCRIPTION OF A HOT STRIP MILL [22,23]

The lay-out of the whole process is shown in Fig. 5.1. The steel slabs are usually stored in the slab yard (1). The slabs are then lifted by the overhead crane to the lifting roller table (4). Then they are pushed to the reheating furnace table by the primary pusher (2). They run on the roller table (4). Then each slab is pushed to the reheating furnace (5) by two pushers (3) in the front of the furnace. These pushers are controlled by control pulpit (Con. 1).

The slab passes through different stages in the furnace until the temperature of the slab reaches the desired value for processing. Then it is pushed out from the furnace to the delivery table (6). The slab will be covered with scales due to the heating. Therefore it passes through the roughing descaler (7) to remove the scales. The descaler consists of two vertical rolls to cause cracks in the scales which cover the slab. The scales are removed by using high pressure water spray while the slab passes through a high-pressure descaling station. This station is controlled by the control pulpit (Con 2). The slab is then transferred from the descaling station or roller table (8).

After that, the thickness of the slab is reduced. This is achieved by passing the slab through the reversing mill forward and backward five times, until it reaches the desired value avoiding the overload of the stand mill. The reversing mill consists of an edging stand (9) and horizontal stand (10). The width of the slab is controlled using the edging stand. The thickness of the slab is reduced using the horizontal stand by controlling the gap between the rollers. The control pulpit (Con 3) controls the reversing mill.

Now, the slab runs over the roller table (11) to the roll table (12) and enters the cropping shear area (13). The front and the tail ends of the slab are cropped using the cropping shear. This is done before entering the slab in the finishing mill.

Then, the slab passes through the finishing descaler (14) which consists of two stages. The first stage is the finishing scale breaker for breaking the scales while the second stage is the hydraulicdescaling station for removing the scales from the slab before entering the finishing mill (15).

The finishing mill is used to reduce the thickness of the slab to the final desired value. The finishing mill consists of six stand in order, three of them are working and the other three are standby. Each stand consists of two rolls. Between each two stands there is a looper (16). The control pulpit (Con 4) controls the finishing mill.

An x-ray gauge meter (17) is used for measuring the output strip thickness.

Then the strip is cooled by water while passing over the run-out roller table (18). This is done to achieve the desired value for coiling temperature. The cooling system is controlled according to strip thickness, temperature, steel grade and structure.

Finally, the strip enters the coiling area (19). The coiling area consists of the coiler drum and four wrapping rollers. The coiler drum is controlled to obtain a constant linear velocity for the strip. Around the coiler drum there are the four wrapping rollers which are used to control the coil shape. Pinch rolls are used to control the coiling tension. The coiling system is controlled by the control pulpit (Con 5).

The final product from this process is either used directly after cutting it to the required shapes, or processed again in the cold rolling mill.

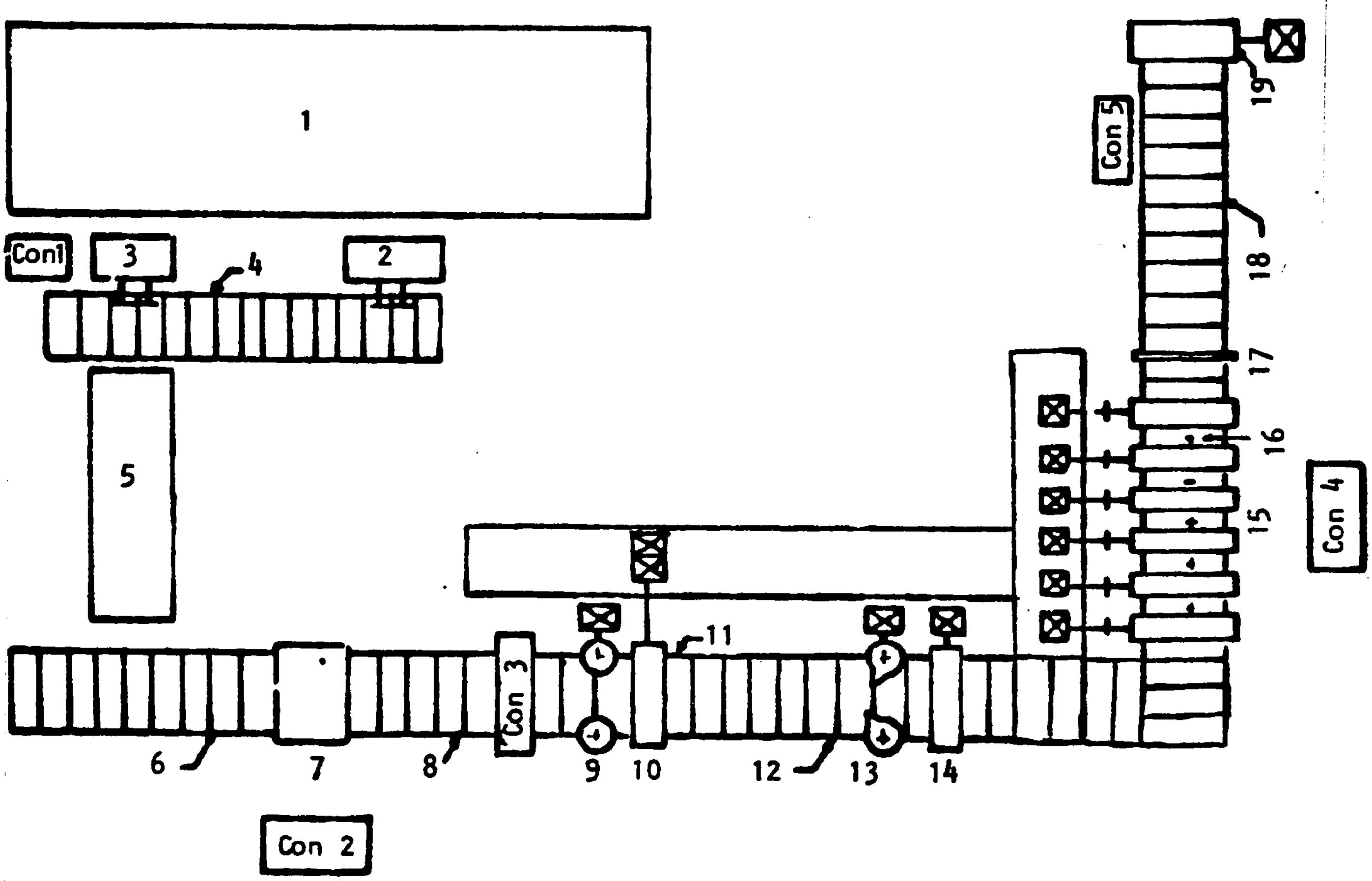

Figure 5.1. The layout of hot rolling finishing mill process.

The finishing mill is selected to be used as practical example where the decentralized controller can be designed using the developed decentralized approaches.

5.3 THE MATHEMATICAL MODEL OF THE FINISHING MILL

In this section, the mathematical model representing the finishing mill, with three stands, which is developed in [22,23] will be presented.

The finishing mill consists of three stands. Each stand has a pair of rolls. The rolls are driven by an armature controlled d.c. motor. The power screw is also driven by an armature controlled d.c. motor. The parameters of a mill stand are shown in Fig. 5.2. The schematic diagram of the finishing mill is shown in Fig. 5.3 where the following notation used in this diagram are

j = number of stands (1, 2, 3)

i, 0, n denote inlet, outlet and neutral plane.

First, the physical variables of the system are organized in nonlinear mathematical equation.

Second, these equations are linearized by the expansion around the operating point. The order of the linearized model is 98. That means if we apply the optimal control theory using quadratic performance measure, we must solve 98 x 98 symmetric Riccati matrix equation equivalent to a system of 4851 scalar equations.

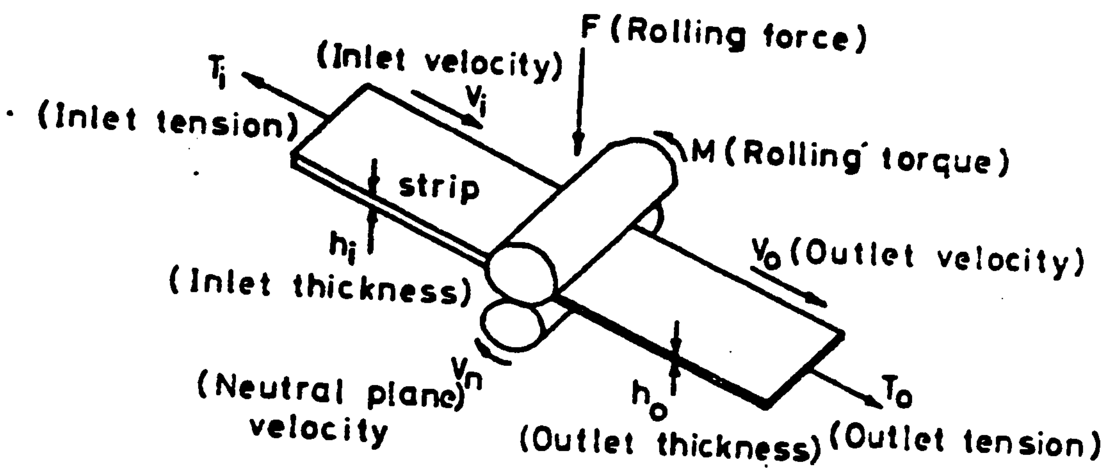

Figure 5.2. The parameters of a mill stand.

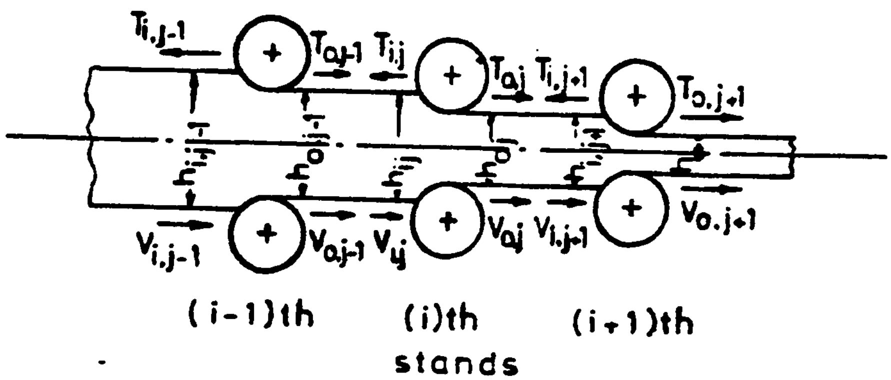

Figure 5.3. The schematic diagram of the finishing mill.

Thus it is necessary to reduce the order of the system in order to reduce the computational effort and the required computer storage.

Third, the model is reduced using the aggregation method mentioned in [1].

The simplified model is represented by the following equation

$$\dot{x} = A\,x + B\,u + Ed$$

where A is a 6 x 6 matrix

and B is a 6 x 3 matrix

and E is a 6 x 2 matrix.

x is the state vector which contains (ω_1, ω_2, ω_3) and (T_{o1}, T_{o2}, T_{o3}) where ω_i is the drive motor angular velocity deviation, and T_{oi} is the strip output tension deviation. u is the control input which contains (e_{a1} , e_{a2}, e_{a3}) where e_{ai} is the armature control voltages.

$$A = \begin{bmatrix} -430. & 0.0 & 0.0 & 4.8 & 0.0 & 0.0 \\ 0.0 & -176.0 & 0.0 & -3.0 & 2.52 & 0.0 \\ 0.0 & 0.0 & -184.0 & -0.91 & -4.025 & 2.5 \\ -640.0 & 375.0 & 0.0 & -97.5 & -3.8 & 0.0 \\ 0.0 & -630.0 & 410.0 & -103.7 & -125.6 & -11. \\ 0.0 & 0.0 & -630.0 & 200.0 & 102.8 & -31.1 \end{bmatrix}$$

$$B = \begin{bmatrix} 3.9 & 0.0 & 0.0 \\ 0.0 & 2.5 & 0.0 \\ 0.0 & 0.0 & 3.7 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \end{bmatrix}$$

$$E = \begin{bmatrix} 0.107 & -3.6 \\ 0.026 & -0.73 \\ -0.0136 & -.38 \\ 2.44 & -43.6 \\ -2.37 & 65.2 \\ 3.0 & -82.0 \end{bmatrix}$$

The value of the disturbance will be taken as $[0.0 \quad 0.3]^T$. This means that there is a step increase in the strip input thickness equal to 0.8 mm.

The performance measure is in the following form:

$$J = \frac{1}{2} \int_0^{\infty} (x^T Q x + u^T R u)\, dt$$

where the values of Q and R matrices are shown below:

$$Q = \begin{bmatrix} 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 5.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 5.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 5.0 \end{bmatrix}$$

$$R = \begin{bmatrix} 1.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 1.0 \end{bmatrix}$$

The choice of Q shows that a larger weight is given for tension variables (T_{o1}, T_{o2}, T_{o3}) w.r.t. the angular speed variables (ω_1, ω_2, ω_3). This is due to the importance of minimizing the tension variations of the strip. The matrix R is chosen so that equal weights are given for the control variables (e_{a1}, e_{a2}, e_3). The initial values for the states will be zero, that means there is no deviation from the operating condition at the beginning.

In order to apply the decentralized control approach, the simplified model equation of the finishing mill will be modified by rearranging the states vector x so that each stand will be considered as a subsystem, with states (ω,T_o) and control e_a shown in Fig. 5.4. The necessary modifications in the A, B and E matrices are made.

The final form of the modified model of the finishing mill will then be as follows:

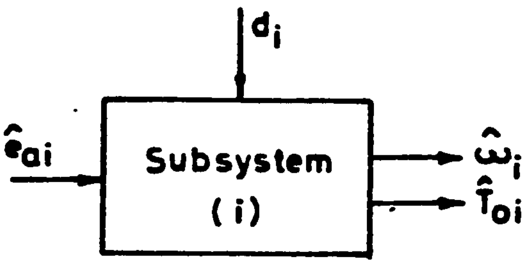

Figure 5.4. The i^{th} subsystem.

$$\dot{x} = A\,x + B\,u + E\,d$$

where x will be $[\omega_1, T_{o1}, \omega_2, T_{o2}, \omega_3, T_{o3}]^T$.

and A, B and E will take the following modified form:

$$A = \begin{bmatrix} -430.0 & 4.8 & 0.0 & 0.0 & 0.0 & 0.0 \\ -640.0 & -97.5 & 375.0 & -3.8 & 0.0 & 0.0 \\ 0.0 & -3.0 & -176.0 & 2.52 & 0.0 & 0.0 \\ 0.0 & -103.7 & -630.0 & -125.6 & 410.0 & -11.0 \\ 0.0 & -0.91 & 0.0 & -4.025 & -184.0 & 2.5 \\ 0.0 & 200.0 & 0.0 & -102.8 & -630.0 & -31.1 \end{bmatrix}$$

$$B = \begin{bmatrix} 3.9 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & 2.5 & 0.0 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 3.7 \\ 0.0 & 0.0 & 0.0 \end{bmatrix}, \quad E = \begin{bmatrix} 0.107 & -3.6 \\ 2.44 & -43.6 \\ 0.026 & -0.73 \\ -2.37 & 65.2 \\ -0.0136 & 0.38 \\ 3.0 & -82.0 \end{bmatrix}$$

5.4 DECOMPOSING THE FINISHING MILL TO THREE SUBSYSTEMS

The model representing the hot rolling finishing mill is decomposed into three subsystems. Each subsystem represents a stand consisting of two rollers. Therefore the decomposition is done on physical basis.

Each subsystem will be described by the following equation:

$$\dot{x}_i = A_i x_i + B_i x_i + B_i u_i + C_i z_i + E_i d_i$$

..... $i = 1,2,3$

where x_i represents the angular speed and the tension of the slab. The input disturbance will be taken as

$$d_i^T = [0.0 \quad 0.3]$$

The performance measure will be also decomposed in the following form:

$$J_i = \frac{1}{2} \int_0^\infty (x_i^T Q_i x_i + u_i^T R_i u_i)\, dt$$

where

$$Q_i = \begin{bmatrix} 1.0 & 0.0 \\ 0.0 & 5.0 \end{bmatrix}, \quad R_i = 1.0$$

----- $i = 1, 2, 3$

The first subsystem matrices are:

$$A_1 = \begin{bmatrix} -430.0 & 4.8 \\ -640.0 & -97.5 \end{bmatrix}, \quad x_1^T = [\omega_1 \quad T_{o1}]$$

$$B_1 = \begin{bmatrix} 3.9 \\ 0.0 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 0.0 & 0.0 \\ 375.0 & -3.8 \end{bmatrix}$$

$$E_1 = \begin{bmatrix} 0.107 & -3.6 \\ 2.44 & -43.6 \end{bmatrix}, \quad z_1^T = [\omega_2 \quad T_{o2}]$$

and the second subsystem matrices are:

$$A_2 = \begin{bmatrix} -176.0 & 2.52 \\ -630.0 & -125.6 \end{bmatrix}, \quad x_2^T = [\omega_2 \quad T_{o2}]$$

$$B_1 = \begin{bmatrix} 2.5 \\ 0.0 \end{bmatrix}, \quad C_2 = \begin{bmatrix} -3.0 & 0.0 & 0.0 \\ -103.7 & 410.0 & -11.0 \end{bmatrix}$$

$$E_2 = \begin{bmatrix} 0.026 & -0.73 \\ -2.37 & 65.2 \end{bmatrix}, \quad z_2^T = [T_{01} \quad \omega_3 \quad T_{o3}]$$

and the third subsystem matrices are:

$$A_3 = \begin{bmatrix} -184.0 & 2.5 \\ -630.0 & -31.1 \end{bmatrix}, \quad x_3^T = [\omega_3 \quad T_{o3}]$$

$$B_3 = \begin{bmatrix} 3.7 \\ \\ 0.0 \end{bmatrix}, \quad C_3 = \begin{bmatrix} -0.91 & -4.025 \\ \\ 200.0 & -102.8 \end{bmatrix}$$

$$E_3 = \begin{bmatrix} -0.0136 & 0.38 \\ \\ 3.0 & -82.0 \end{bmatrix}, \quad z_3^T = [T_{o1} \quad T_{o2}]$$

5.5 OPTIMAL CONTROLLER

The optimal control takes the form:

$$u = -R^{-1} B^T P x + v$$

where the steady state value of Riccati matrix P is determined as:

$$P = \begin{bmatrix} 0.18 & -0.12 & -0.08 & -0.05 & 0.16 & -0.07 \\ -0.12 & 0.09 & 0.05 & 0.03 & -0.12 & 0.05 \\ -0.08 & 0.05 & 0.32 & -0.06 & -0.10 & -0.01 \\ -0.05 & 0.03 & -0.06 & 0.05 & -0.04 & 0.03 \\ 0.16 & -0.12 & -0.10 & -0.04 & 0.24 & -0.10 \\ -0.07 & 0.05 & -0.0 & 0.03 & -0.10 & 0.06 \end{bmatrix}$$

and

$$v^T = [-0.257 \quad 0.024 \quad -0.192]$$

Then, the controls inputs are

$$u_1 = [0.7 \quad -0.5 \quad -0.3 \quad -0.2 \quad 0.6 \quad -0.3]\, x - 0.257$$

$$u_2 = [-0.2 \quad 0.1 \quad 0.7 \quad -0.1 \quad -0.2 \quad -0.02]\, x + 0.02$$

$$u_3 = [0.6 \quad -0.4 \quad -0.4 \quad -0.2 \quad 0.9 \quad -0.4]\, x - 0.192$$

Simulation results showing state trajectories and control functions are given in Fig. 5.5.

5.6 DECENTRALIZED MODEL-FOLLOWING CONTROLLER

Formulating the problem into the model-following form requires the designs to choose the crude model. The chosen crude models are

$$A_{z1} = \begin{bmatrix} -176.0 & 2.52 \\ -630. & -125.6 \end{bmatrix}$$

$$A_{z2} = \begin{bmatrix} -97.5 & 0.0 & 0.0 \\ -0.91 & -184. & 2.5 \\ 200. & -630. & -31.1 \end{bmatrix}$$

$$A_{z3} = \begin{bmatrix} -97.5 & -3.8 \\ -103.7 & -125.6 \end{bmatrix}$$

The gain matrices are calculated and the resultant gain matrices for each controller are:

$$G_{1x} = [0.172 \quad -0.113]$$
$$G_{1z} = [-0.202 \quad 0.0001]$$

$$K_1 = \begin{bmatrix} -0.254 & 0.25 \\ 0.003 & -0.003 \end{bmatrix}$$

$$G_{2x} = [0.352 \quad -0.0097]$$
$$G_{2z} = [0.09 \quad -0.239 \quad 0.009]$$

$$K_2 = \begin{bmatrix} 0.079 & -0.055 \\ -0.35 & 0.229 \\ 0.009 & -0.006 \end{bmatrix}$$

$$G_{3x} = [2.072 \quad -0.615]$$
$$G_{3z} = [-1.472 \quad 0.446]$$

$$K_3 = \begin{bmatrix} -0.565 & 0.272 \\ 0.251 & -0.127 \end{bmatrix}$$

The equations of the firstr subsystem controller are:

$$\dot{z}_{m1} = A_{z1} z_{m1}$$

$$\dot{x}_{m1} = A_{o1} x_{m1} + C_{o1} z_{m1} + W_1$$

$$x_{e1} = x_1 - x_{m1}$$

$$z_{e1} = -K_1 x_{e1}$$

$$z_1 = z_{m1} + z_{e1}$$

$$u_1 = -G_{1x} x_1 - G_{2z} z_1 + v_1$$

where

$$A_{o1} = \begin{pmatrix} -430.67 & 5.239 \\ -640.00 & -97.5 \end{pmatrix}$$

$$C_{o1} = \begin{pmatrix} 0.789 & -0.0002 \\ 375.0 & -3.8 \end{pmatrix}$$

$$W_1^T = [-1.1519 \ \ -13.08]$$

$$v_1 = -0.0184$$

The equations for the second subsystem controller are:

$$\dot{z}_{m2} = A_{z2} \quad z_{m2}$$

$$\dot{x}_{m2} = A_{o2}\ x_{m2} + C_{o2}\ z_{m2} + W_2$$

$$x_{e2} = x_2 - x_{m2}$$

$$z_{e2} = -K_2\, x_{e2}$$

$$z_2 = z_{m2} + z_{e2}$$

$$u_2 = -G_{2x}\, x_2 - G_{2z}\, z_2 + v_2$$

where

$$A_{o2} = \begin{bmatrix} -176.88 & 2.762 \\ -630.0 & -125.6 \end{bmatrix}$$

$$C_{o2} = \begin{bmatrix} -3.226 & 0.598 & -0.022 \\ -103.7 & 410.0 & -11.0 \end{bmatrix}$$

$$W_2^T = [-0.13 \quad 19.56]$$

$$v_2 = \quad 0.03539$$

The equations for the third subsystem controller are:

$$\dot{z}_{m3} = A_{z3}\, z_{m3}$$

$$\dot{x}_{m3} = A_{o3}\, x_{m3} + C_{o3}\, z_{m3} + W_3$$

$$x_{e3} = x_3 - x_{m3}$$

$$z_{e3} = -K_3\, x_{e3}$$

$$z_{e3} = -G_{3x}\, x_3 - G_{3z}\, z_3 + v_3$$

where

$$A_{03} = \begin{bmatrix} -191.67 & 4.774 \\ -630.0 & -31.1 \end{bmatrix}$$

$$C_{o3} \quad \begin{bmatrix} 4.535 & & -5.673 \\ 200.0 & -102.8 & \end{bmatrix}$$

$$W_3^T = \quad [-1.5427 \;\; -24.6]$$

$$v_3 = \quad -0.4478$$

Simulation results showing the state trajectories and the control functions are given in Fig. 5.5.

5.7 MODIFIED DECENTRALIZED MODEL-FOLLOWING CONTROLLER

After putting the problem in the modified model-following form, the gain matrices are calculated. The resultant gain matrices for each controller are:

$$G_{1x} = [0.015 \quad -0.0103]$$

$$G_{1z} = [-0.0098 \quad 0.0001]$$

$$K_{1x} = \begin{bmatrix} -0.993 & 1.985 \\ 0.0101 & -0.0201 \end{bmatrix}$$

$$K_{1z} = \begin{bmatrix} 0.7302 & -0.006 \\ -0.007 & 0.0001 \end{bmatrix}$$

$$G_{2x} = [0.0218 \quad -0.006]$$

$$G_{2z} = [0.003 \quad -0.0099 \quad 0.0003]$$

$$K_{2x} = \begin{bmatrix} 0.24 & -0.5 \\ -1.05 & 1.9 \end{bmatrix}$$

$$K_{2z} = \begin{bmatrix} 0.04 & -0.2 & 0.005 \\ -0.2 & 0.7 & -0.02 \\ 0.005 & -0.02 & 0.0005 \end{bmatrix}$$

$$G_{3x} = [0.082 \quad , \quad -0.032]$$

$$G_{3z} = [-0.037 \quad , \quad 0.0136]$$

$$K_{3x} = \begin{bmatrix} -1.73 & 1.875 \\ 0.79 & -0.925 \end{bmatrix}$$

$$K_{3z} = \begin{bmatrix} 0.685 & -0.302 \\ -0.307 & 0.1387 \end{bmatrix}$$

The equations required to implement the controller for each subsystem controller are:

<u>Subsystem 1:</u>

$$\dot{z}_{m1} = A_{z1} z_{m1}$$

$$z_1 = -K_{1x} x_1 - K_{1z} z_{m1} + v_{z1}$$

$$u_1 = -G_{1x} x_1 - G_{1z} z_{m1} + v_{u1}$$

where

$$v_{z1} = [0.0293 \qquad -0.0003]^T$$

$$v_{u1} = -0.0005$$

Subsystem 2:

$$\dot{z}_{m1} = A_{z2}\, z_{m2}$$

$$z_2 = -K_{2x}\, x_2 - K_{2z}\, z_{m2} + v_{z2}$$

$$u_2 = -G_{2x}\, x_2 - G_{2z}\, z_{m2} + v_{u2}$$

where

$$v_{z2} = [0.0087 \qquad -0.039 \qquad 0.001]^T$$

$$v_{u2} = 0.001$$

Subsystem 3:

$$\dot{z}_{m3} = A_{z3}\, z_{m3}$$

$$z_3 = -K_{3x}\, x_3 - K_{3z}\, z_{m3} + v_{z3}$$

$$u_2 = -G_3\, x_3 - G_{3z}\, z_{m3} + v_{u3}$$

where

$$v_{z3} = [0.0932 \quad , \quad -0.0404]^T$$

$$v_{u3} = -0.0062$$

It should be noted here, that the crude model is the same as the one used in the previous approach, (Section 5.6). Simulation results showing the state trajectories and the control functions are given in Fig. 5.5 in offline and online cases.

5.8 DECENTRALIZED INTERACTION-REJECTION CONTROLLER

Ignoring the interaction between the subsystems, the local optimal controllers take the form:

$$u_{11} = [0.1719 \quad , \quad -0.1126]\ x_1 - 1.08$$

$$u_{12} = [0.3514 \quad , \quad -0.0965]\ x_2 + 0.0354$$

$$u_{13} = [2.0631 \quad , \quad -0.6126]\ x_3 - 0.4428$$

The disturbance compensating component of the control function is determined, for each subsystem from the following equations:

$$\dot{u}_{21} = 0.000$$

$$\dot{u}_{22} = -50.7\, u_{22} - [0.000014 \quad -0.00]\, x_2 - [0.00001 \; - \; 0.000012 \quad 0.00]\, z_2 + 0.00012$$

$$\dot{u}_{23} = -163.435\, u_{23} - [0.08 \; - \; 0.02]\, x_3 - [-0.07 \quad 0.0186]\, z_3 - 0.0284$$

The final form of the controls will be

$$u_1 = u_{11} + u_{21}$$

$$u_2 = u_{12} + u_{22}$$

$$u_3 = u_{13} + u_{23}$$

where

$$\dot{z}_1 = (A_{z1} - G_{1z})\, z_1 - G_{1x}\, x_1 + \eta_1$$

$$\dot{z}_2 = (A_{z2} - G_{2z})\, z_2 - G_{2x}\, x_2 + \eta_2$$

$$\dot{z}_3 = (A_{z3} - G_{3z})\, z_3 - G_{3x}\, x_3 + \eta_3$$

The values of the previous-mentioned matrices are:

A_{z1}, A_{z2}, and A_{z3} have the same values as given in Section (5.6).

$$G_{1x} = \begin{bmatrix} -0.052 & 0.03 \\ 0.0 & 0.0 \end{bmatrix},$$

$$G_{1z} = \begin{bmatrix} 0.0682 & 0.0001 \\ 0.0001 & 0.0 \end{bmatrix},$$

$$G_{2x} = \begin{bmatrix} 0.036 & -0.009 \\ -0.0956 & 0.0262 \\ 0.0035 & -0.0009 \end{bmatrix},$$

$$G_{2z} = \begin{bmatrix} 0.0105 & -0.0256 & 0.001 \\ -0.0256 & 0.0667 & -0.0025 \\ 0.001 & -0.0025 & 0.0001 \end{bmatrix},$$

$$G_{3x} = \begin{bmatrix} -0.3957 & 0.1087 \\ 0.1199 & -0.0346 \end{bmatrix},$$

$$G_{3z} = \begin{bmatrix} 0.3097 & -0.0889 \\ -0.0889 & 0.0263 \end{bmatrix},$$

$$\eta_1^T = [0.0075 \quad 0.0001],$$
$$\eta_2^T = [0.0057 \quad -0.0111 \quad 0.0006],$$

$$\eta_3^T = [0.1146 \quad -0.0293]$$

Simulation results showing the state trajectories and the control functions are given in Fig. 5.5.

5.9 COMMENTS ON THE SIMULATION

1. The numerical methods used for the simulation have a big effect on the results. Many methods of integration are tried in the simulation and the best one used is Fourth-order Runge-Kutta method in CSMP Computer Programs Package.

2. The state trajectories obtained from the off-line modified decentralized controller has the smallest offset.

3. In the case of the modified decentralized model-following controller, the control signals is the minimum in both cases offline and online.

4. The performance measure in the case of the modified decentralized model-following controller (online) is the best as compared to the other approaches excluding the optimal case, of course. The performance measure values for all the approaches are given in Table III.

5. From the simulation results showed the potential of the modified decentralized controller.

6. Simulation is also carried out when the system is subjected to a random disturbance (± 30%). The simulation results showing the state trajectories are given in Fig. 5.6. The best approach accommodating this variation is the modified decentralized model-following one.

7. From Fig. 5.5, it can be noticed that the modified model-following approach in the offline case has a good properties which can be used in the online case such as the interactions.

TABLE III. The Performance Measure when Different Approaches are Applied to the Practical Application.

The approach	The performance measure
Optimal	6.7038E-3
Model-Following	5.4177E-2
Modified Model-following (Online)	1.8639E-2
Interaction-Rejection	4.2543E-2

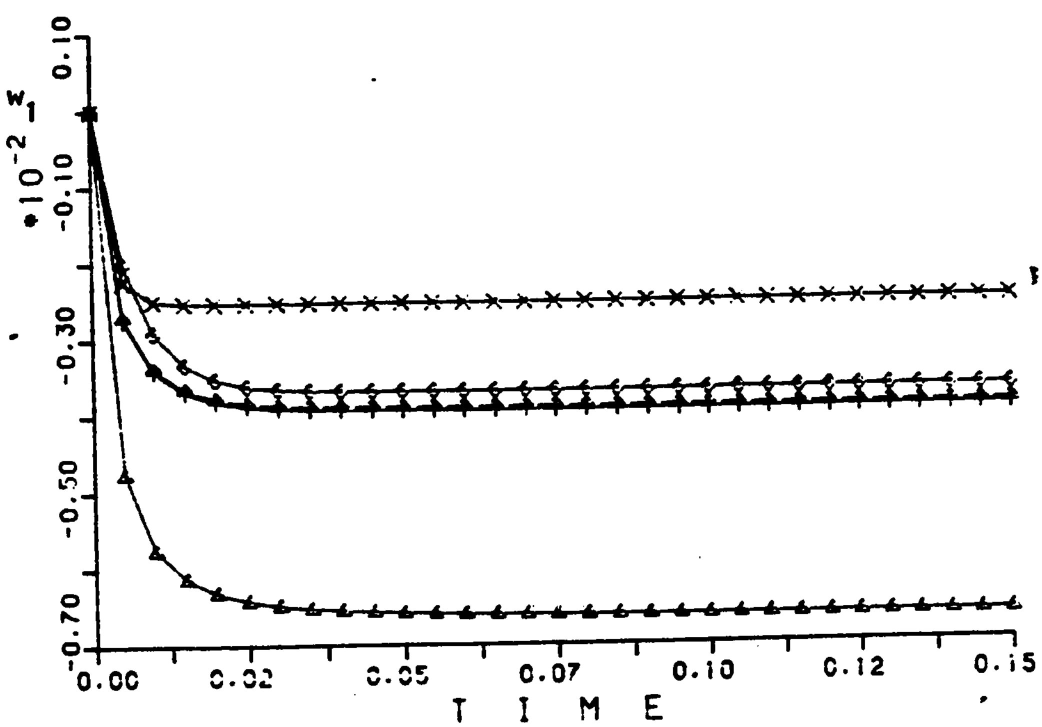

Figure 5.5(a). The angular velocity deviation (w_1) trajectory.

Δ	optimal
+	model-following
x	modified model-following (offline)
◊	modified model-following (online)
♣	Interaction-rejection

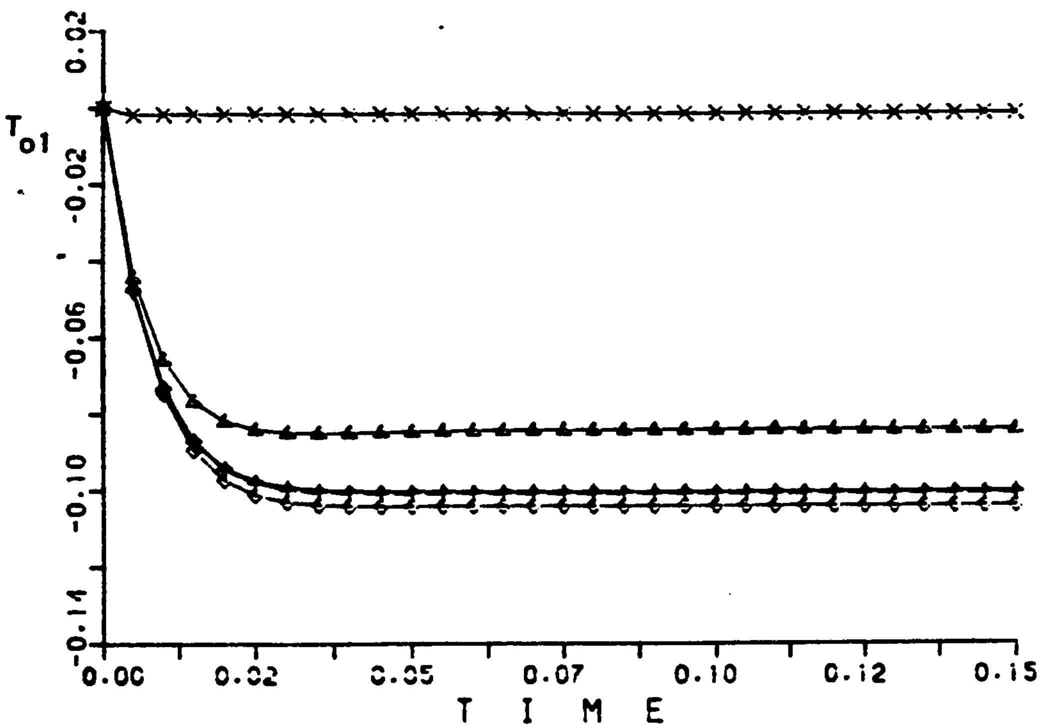

Figure 5.5(b). The tension deviation (T_{o1}) trajectory.

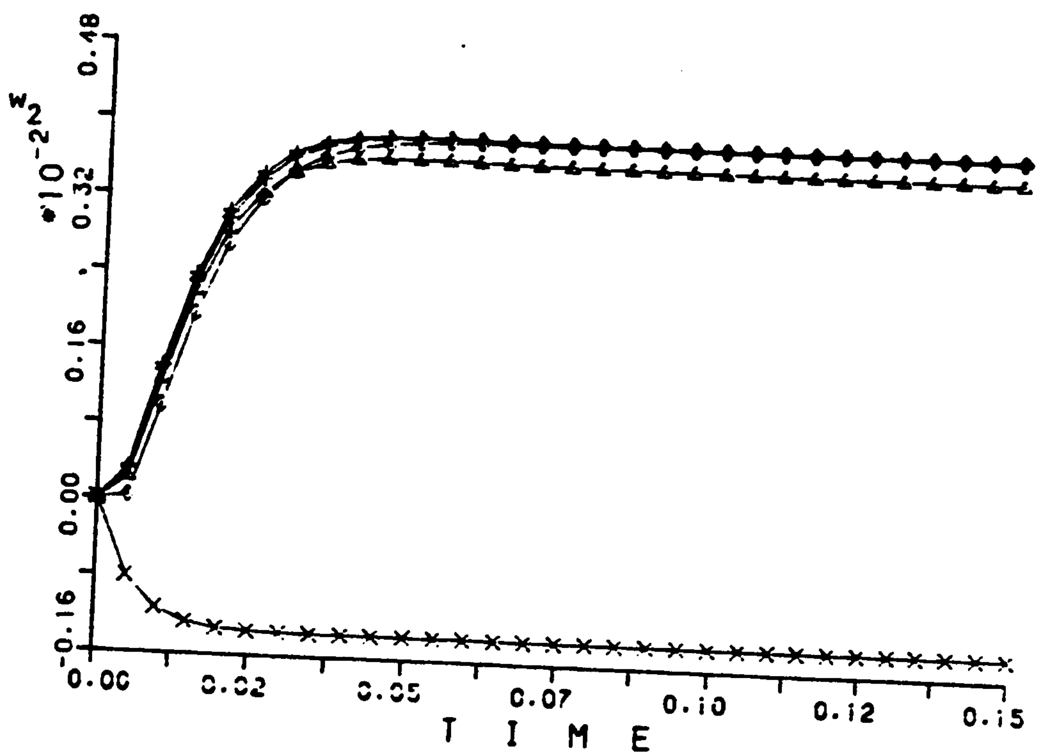

Figure 5.5(c). The angular velocity deviation (w_2) trajectory.

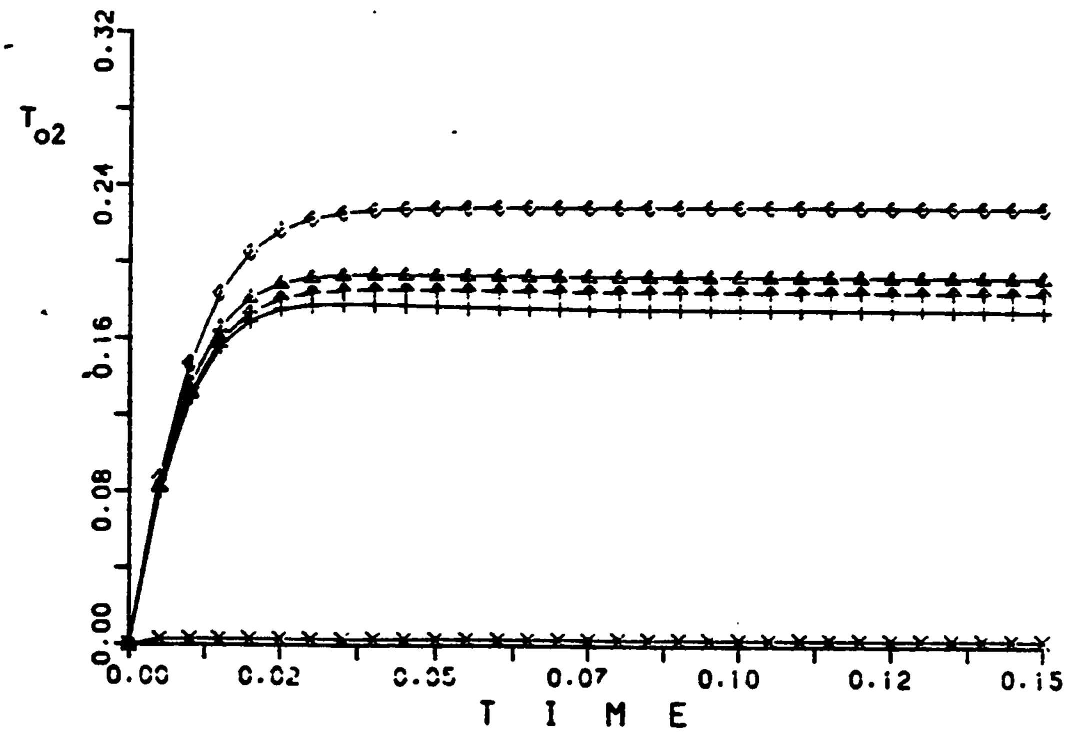

Figure 5.5(d). The tension deviation (T_{o2}) trajectory.

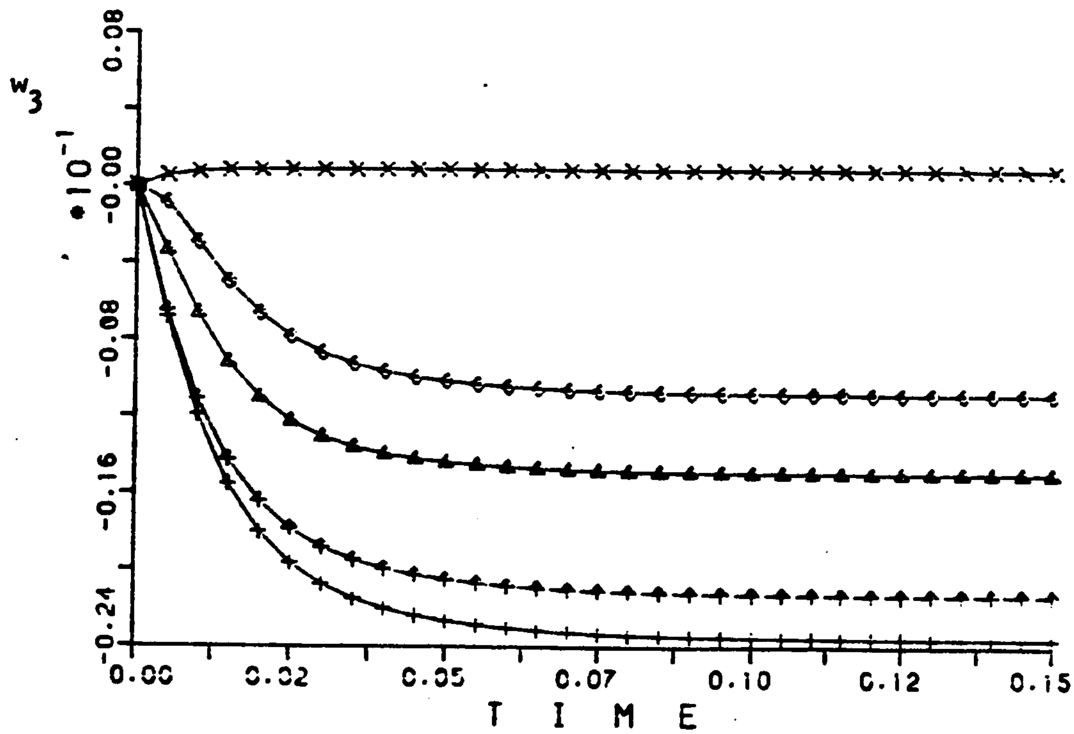

Figure 5.5(e). The angular velocity deviation (w_3) trajectory.

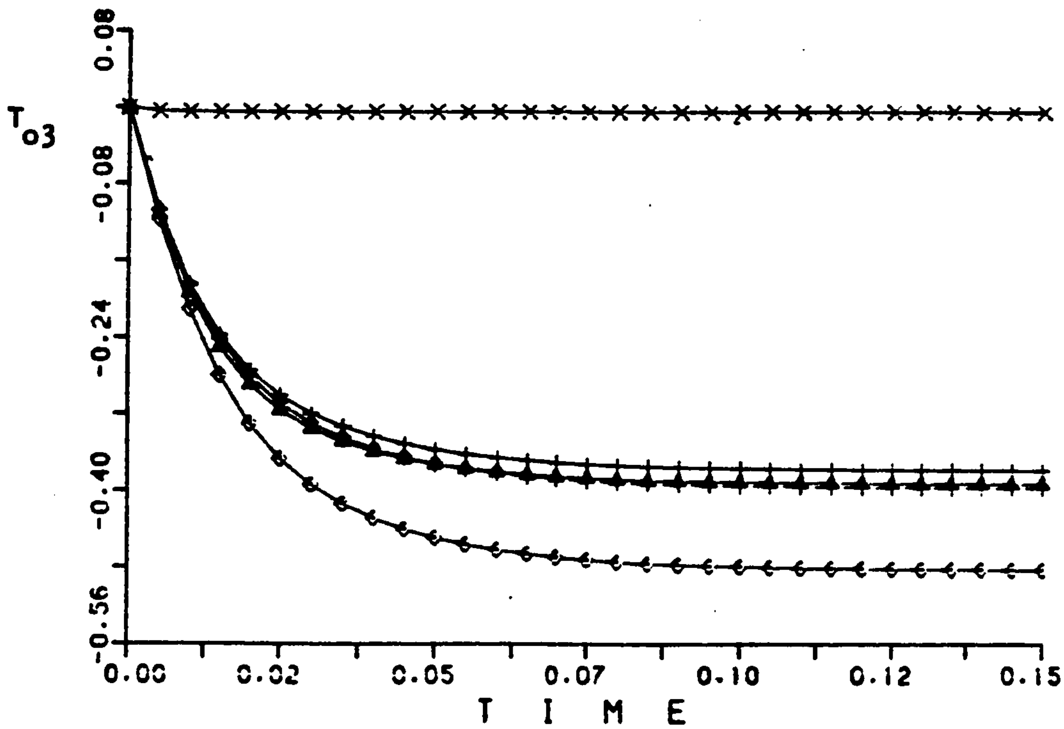

Figure 5.5(f). The tension deviation (T_{o3}) trajectory.

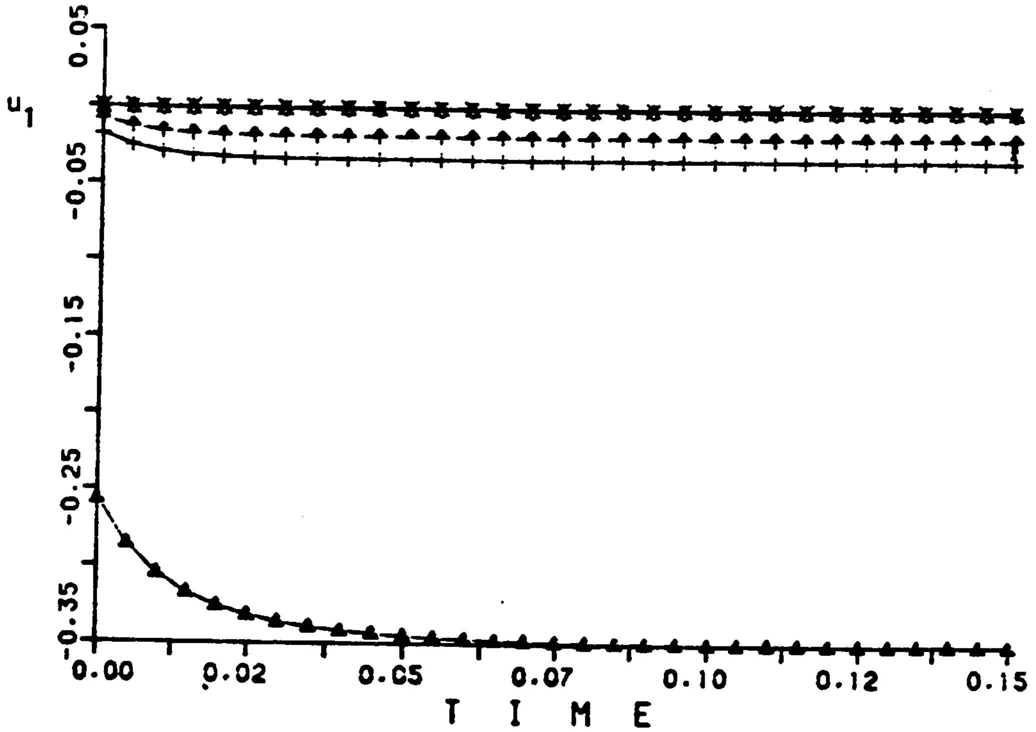

Figure 5.5(g). The control signal $u_1(e_{ai})$.

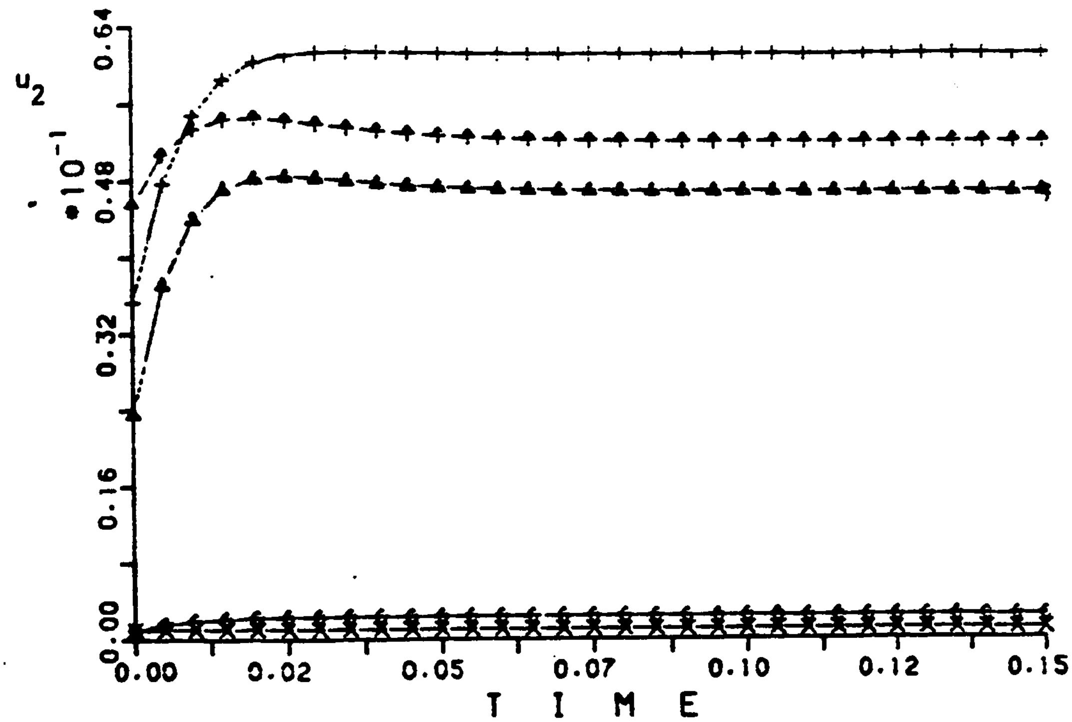

Figure 5.5(h). The control signal u_2 (e_{a2}).

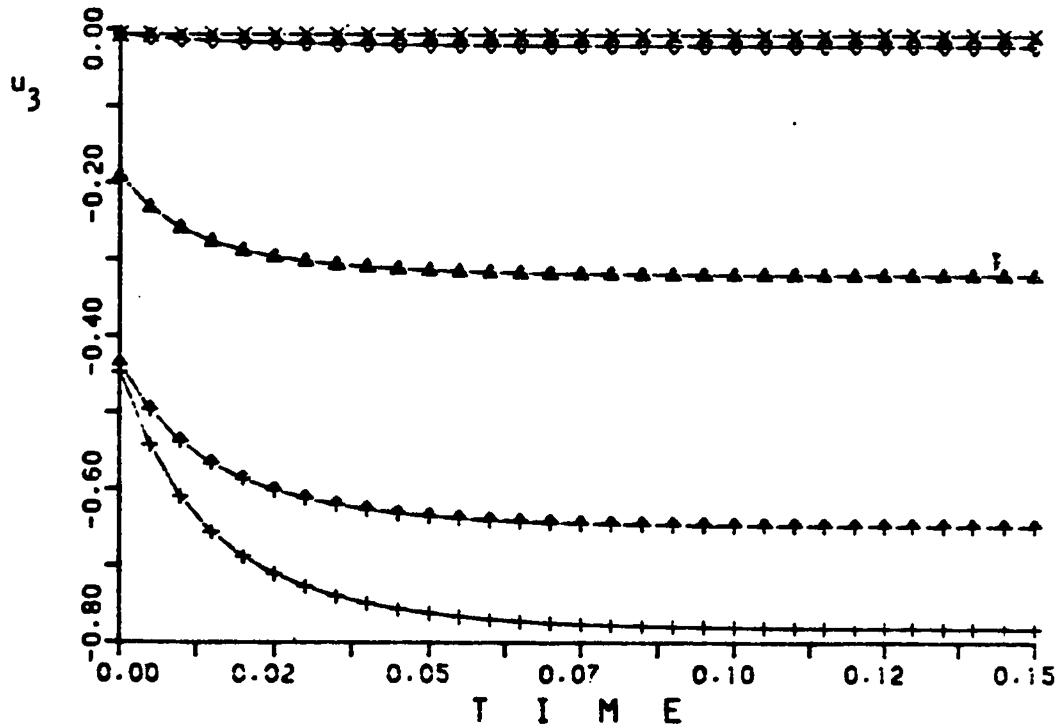

Figure 5.5(i). The control signal u_3 (e_{a3}).

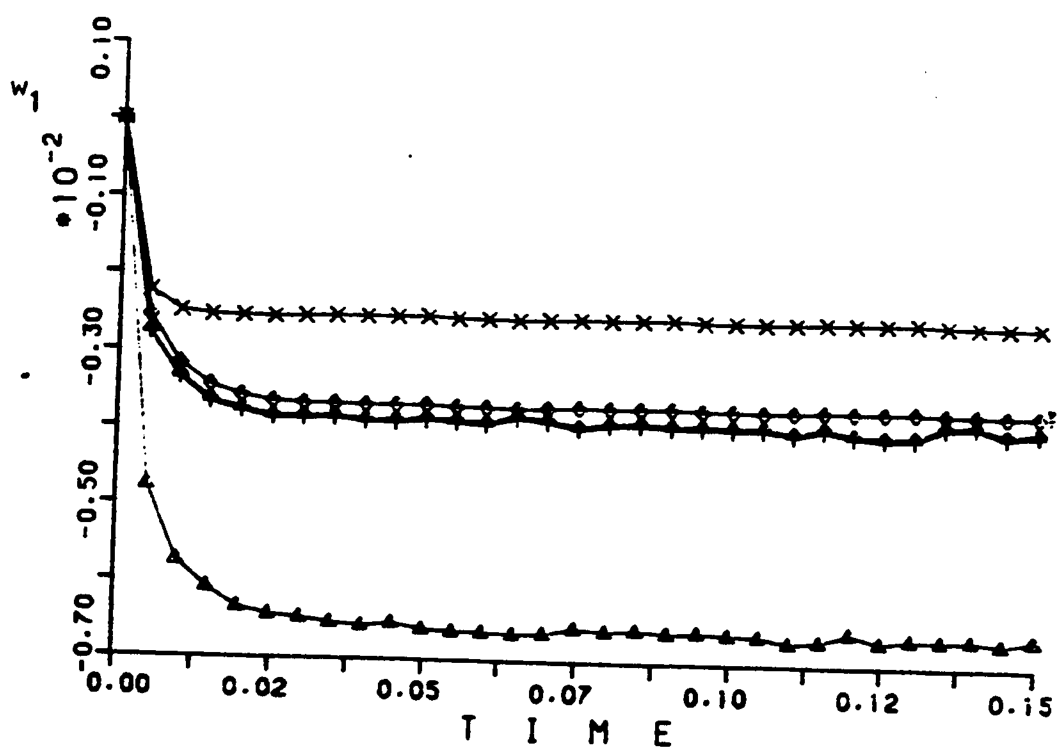

Figure 5.6(a). The angular velocity deviation (w_1) trajectory.

Δ	optimal
+	model-following
x	modified model-following (offline)
◊	modified model-following (online)
◆	Interaction-rejection

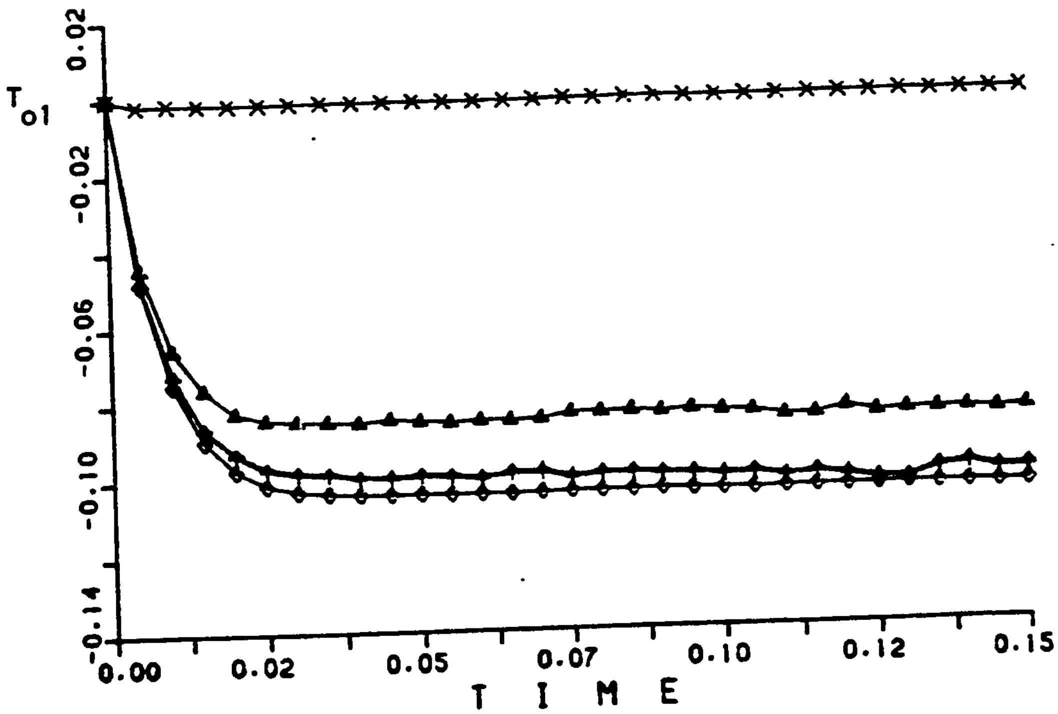

Figure 5.6(b). The tension deviation (T_{o1}) trajectory.

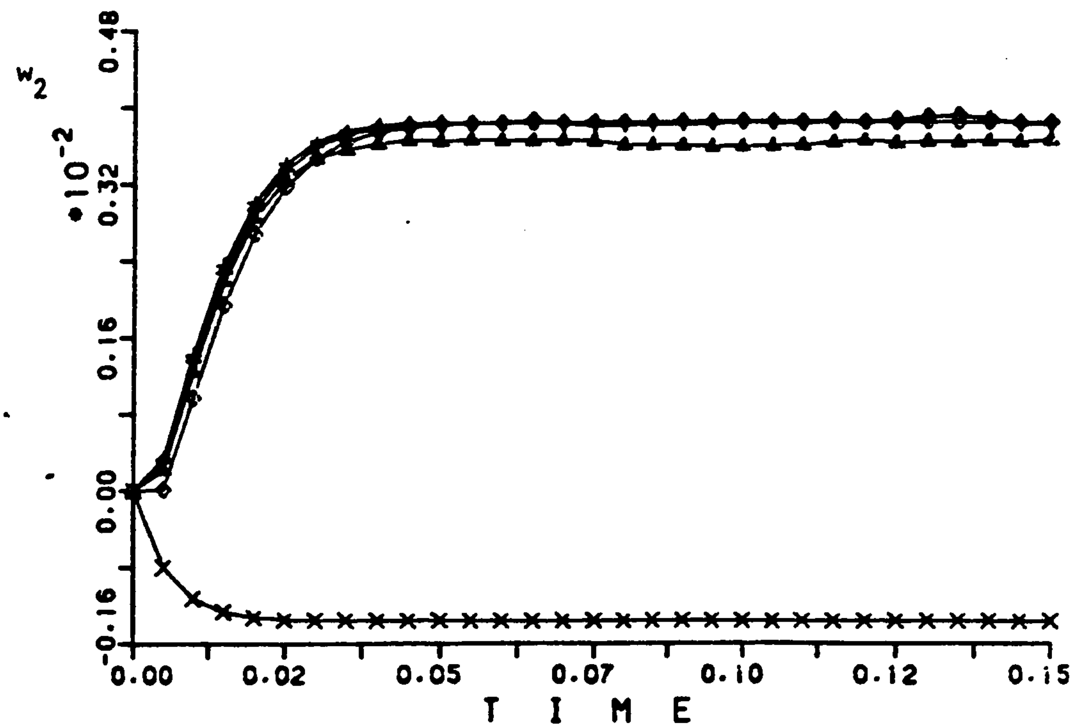

Figure 5.6(c). The angular velocity deviation (w_2) trajectory.

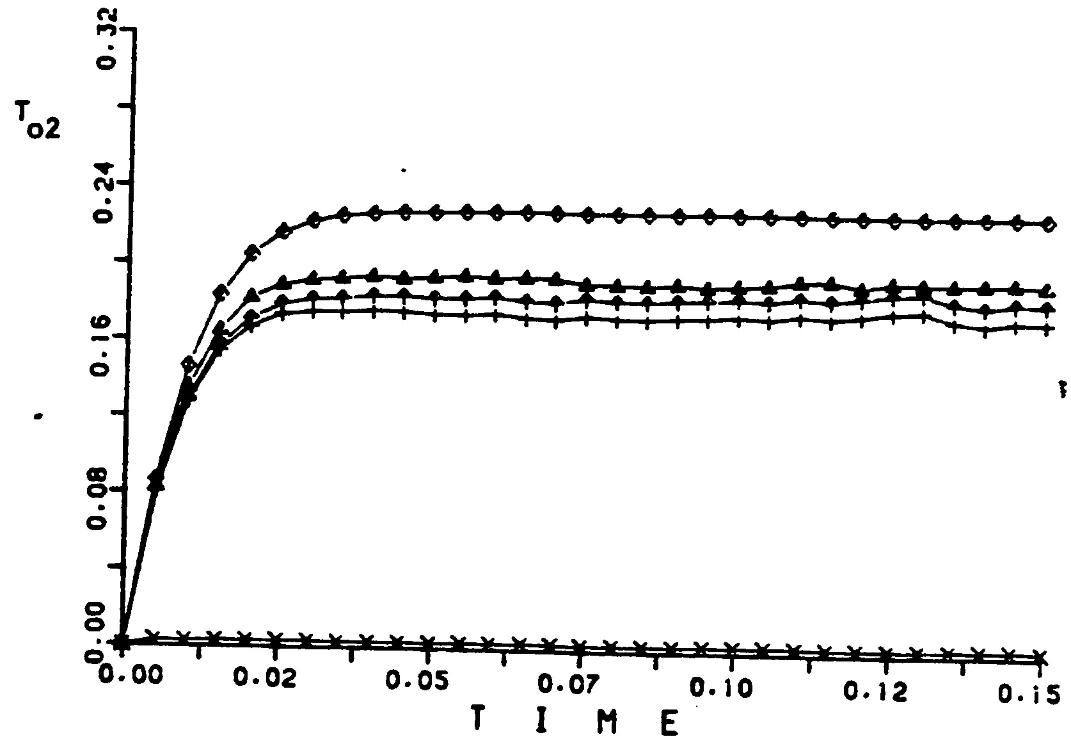

Figure 5.6(d). The tension deviation (T_{o2}) trajectory.

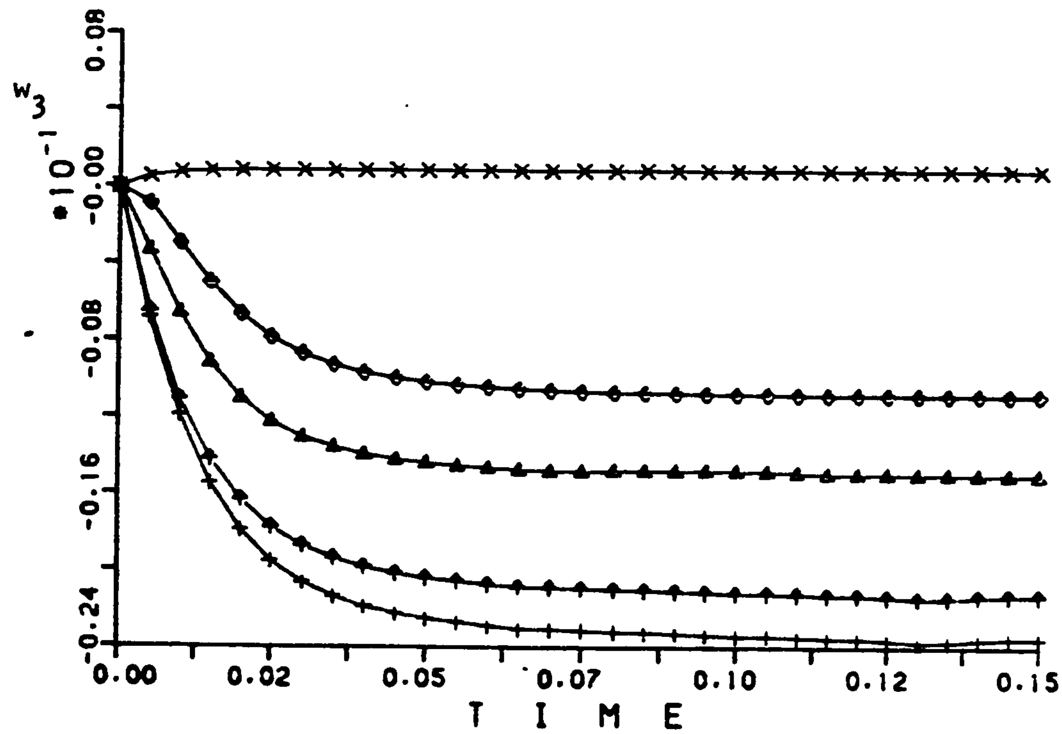

Figure 5.6(e). The angular velocity deviation (w_3) trajectory.

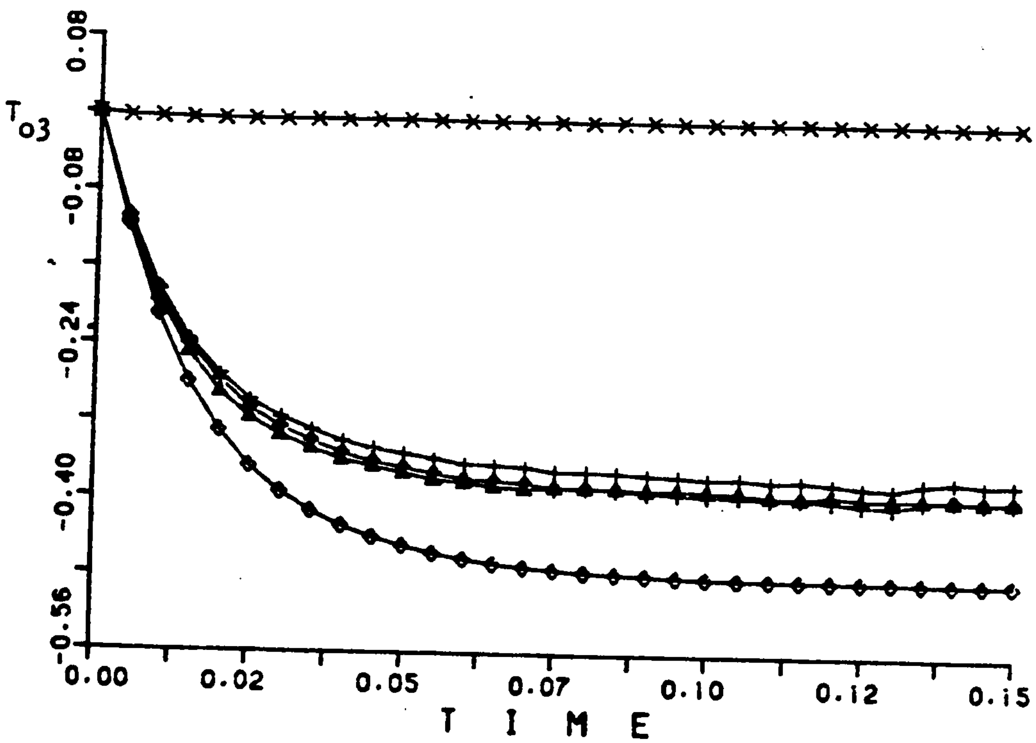

Figure 5.6(f). The tension deviation (T_{o3}) trajectory.

6. CONCLUSION AND RECOMMENDATIONS

6.1 CONCLUSION

The decentralized control problem of interconnected dynamical systems has been considered under the assumption that a linear mathematical model of the interconnected dynamical system is available in state space form. In these methods of design, the interaction between subsystems not available to the controller are obtained from a low-order crude model appropriately selected. The trajectories of the interaction vector are improved online before being used to generate the appropriate control signal.

The decentralized model-following approach proposed in [6,7] is considered with a modification to accommodate a known disturbance. Another modified decentralized model-following approach is also developed. In this approach if we assume a zero initial condition for the interaction then the control law will depend only on the states of the system and in this case it will be very simple to implement. This assumption is valid because the effect of the change of initial condition of the interaction in the states is negligible.

In this approach, we achieved the best performance measure and a good state response with a minimum control effort. As a conclusion, the modified decentralized model-following controller is easy to design and can be worked out subsystem by subsystem. No information transfer between subsystems is necessary.

This results in a low implementation cost of the controller. Moreover the controller can be implemented using microprocessors.

6.2 RECOMMENDATIONS

It is recommended that an efficient computer program package is developed for simulation purposes with graphics capabilities, that will help in this type of studies.

6.3 TOPICS FOR FURTHER RESEARCH

1. Method to calculate the best crude model for the interaction.

2. Decentralized controller design using implicit model-following technique.

3. Decentralized controller using stochastic approach techniques for interaction rejection, when the system is subjected to random disturbance input.

4. Combining the offline and online methods in the modified decentralized model-following controller.

5. Study of the stability of the global system using the modified decentralized model-following controller.

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